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Year Five Pupils' Understanding of Relationship Between Addition and Subtraction

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Abstract: Conceptual understanding of properties of operations is an important element of algebraic thinking in primary school. Mathematical processes should be focused rather than mathematical products starting from primary school. The purpose of this study was to examine the Year Five pupils' understanding of relationship between addition and subtraction. Researchers utilized quantitative approach to investigate Year Five pupils' conceptual understanding of addition and subtraction. Pencil and paper-based assessment consisting of three items was employed to collect the data. The three items comprised direction of change and relationship between addition and subtraction items. The three items also consist of reasoning sections. This article reports the analysis of the responses of 720 Year Five pupils from a district of Malacca. The findings showed the majority of the sample were unable to perform well for the items testing relationship between addition and subtraction. They could not work with addition and subtraction properties. The majority of them were also unable to provide conceptual reasoning for their answer. Only about half of the sample were aware about the inverse relationship of addition and subtraction.

Keywords: *Arithmetic Generalization, Algebraic Thinking, Early Algebra, Properties of Operations*

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Introduction

Arithmetic generalisation plays an important role in developing algebraic thinking (Carpenter, Franke, & Levi, 2003; Carraher, Schliemann, Brizuela, & Earnest, 2006; Haldar, 2014; Kaput, 2008). Arithmetic generalisation involves understanding of properties involved in arithmetic. For example, understanding of arithmetic identities and inverse relations (Haldar, 2014). Generalisation is the

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fundamental key to acquire algebraic thinking in early age. It is often assumed that arithmetic deals with operations involving specific numbers while algebra deals with variables and functions (Carraher et al., 2006). This assumption creates a cognitive gap between arithmetic and algebra.

There must be a smooth transition from arithmetic to algebra (Warren, 2003). Primary schools often focused on mathematics strategies more than mathematical processes (Malara & Navarra, 2003; Mason, 2008). For instance, while teaching arithmetic, properties of operations should be highlighted (Slavit, 1999). Instead of focusing on the correctness of solution, classroom discussions should encourage children to think beyond getting a correct solution (Carpenter et al., 2003). Properties such as sum of odd numbers will be an even number, any number added to a zero will remain unchanged and addition increases the value and subtraction decreases the value are some examples of algebraic thinking elements that should be focused while teaching arithmetic in primary school mathematics classrooms.

Many studies have been conducted to examine the arithmetic generalization capabilities of young children (McNeil & Alibali, 2005; Rittle-Johnson & Alibali, 1999; Van Amerom, 2003). Teaching experiment conducted by McNeil and Alibali (2005) showed that young children as early as seven years old are able to look at the number sentence as a whole and generate correct strategy to solve it. For example, when given a number sentence such as 7 + 4 + 5 = 7 + __, children who received equal sign concept lessons were able to generate correct strategy by adding 4 and 5 to find the answer in the blank. The same number sentence can be solved by finding answer on left hand side and then subtract 7 from the total to find the correct solution (16 - 7 = 9). This method also yields the correct answer. However, the latter method did not encourage students to think beyond computation and make use of equal sign and the properties of operations. As a consequence, primary pupils face inflexibility in the knowledge of operational patterns and failed to develop new strategies when exposed to novel equations.

According to Slavit (1999), properties of operations help smooth transition from arithmetic to algebra. He provided evidence by examining development of a student from first grade to fourth grade. The findings provided evidence for the development of algebraic ways of thinking and use of addition on unknown and arbitrary quantities. In this regard, this study investigates Year Five pupils' understanding regarding relationship between addition and subtraction. These pupils are 11 years old. Year Five pupils definitely will not have problems doing computations involving addition and subtraction as the addition and subtraction lessons have been incorporated in the syllabus since year one. However, it is questionable to what extent they understand about the properties of addition and subtraction.

The purposes of this study are as follows.

To investigate Year Five pupils' understanding on relationship between addition and subtraction.

To investigate Year Five pupils' understanding on direction of change involving addition and subtraction.

To investigate Year Five pupils' reasoning on the relationship and direction of change involving addition and subtraction.

In line with these purposes of the study, the following research items were formulated.

What is the Year Five pupils' understanding on relationship between addition and subtraction?

What is the Year Five pupils' understanding on direction of change involving addition and subtraction?

How is the Year Five pupils' reasoning on the relationship and direction of change involving addition and subtraction?

Methodology

This is a quantitative study. It utilized descriptive research design. The purpose of this study is not to find the reasons behind year five pupils' understanding of relationship between addition and subtraction.

It was aimed to obtain information concerning the current status of year five pupils' understanding or in other word the existing knowledge on relationship between addition and subtraction. Thus, descriptive research design well suited the purpose of this study.

Sample

The sample of this study were from National Schools in a district of Malacca in Malaysia. We obtained a list of schools in the district of Malacca from the Ministry of Education. Then the schools were randomly chosen using spread sheet software. In this study, Microsoft Excel spreadsheet was used to make a random selection of schools. Random integer generation function was used in the spread sheet software in order to select a random school from the lists. Entire Year Five pupils were involved in this study from the chosen schools. A total of 720 Year Five pupils took part in this study. The samples comprised 370 (51.4%) female and 350 (48.6%) male pupils.

Instrument

The data were collected using tasks adapted from Haldar (2014). Total three items were used (see Appendix). All the three items were presented in English and Malay languages to prevent language factor from influencing the results. The three items were scored dichotomously. Question 1 aimed at testing the pupils' ability to identify the inverse property of addition and subtraction. Adding and subtracting a same number would leave the initial number unchanged. Question 2 and 3 aimed at investigating the relationship between addition and subtraction. Question 2 is aimed at testing the pupils' understanding on adding more than what is substracted yields a number that is greater than the initial number. While question 3 tests the understanding on subtracting more than what is added yields a number that is less than the initial number. All three items come with four multiple choice answers. Participants are required to choose answers from A, B, C and D.

Data Collection and Analysis

The data for this study were collected using pencil and paper based assessment. The data reported in this study is part of a larger study. The participants of this study were given an assessment and required to finish it within an hour. As explained in the previous section, each question has two sections. The first section requires participants to choose an answer and the second section requires them to select an appropriate reasoning for the answer chosen. Each question has a reasoning section which requires student to choose an appropriate explanation for the answer chosen. The reasoning choices comprised conceptual, procedural and incorrect reasoning. The reasoning sections were scored from 0 to 2. Selection of conceptual, procedural and incorrect reasoning carries 2, 1 and 0 score respectively. The maximum score for the first section is 1. The maximum score for second section is 2. Thus each question's maximum score is 3. The data were keyed in and analyzed using IBM SPSS (v22.0) software. Descriptive analysis was used to answer the research items of this study.

Results and Discussion

This section discusses the findings related to performance of Year Five pupils for items involving relationship between addition and subtraction. The results are displayed in Table 1. Overall 450 (62.5%) participants managed to score correct in question 1. Some 338 (46.9%) participants scored correct for question 2, while 352 (48.0%) participants scored correctly for question 3. These results demonstrated the participants scored moderately for question 1 and below average for both question 2 and question 3.

Table 1

Participants' correct responses for the three items

	Frequency	**Percentage**
Question 1	450	62.5
Question 2	338	46.9
Question 3	352	48.9

Table 2 shows the types of reasoning chosen by the participants for each question. For question 1, 52.6% of the participants were able to provide conceptual reasoning for inverse operations. Some 36.0% of the participants gave procedural reasoning while 11.4% of the participants gave incorrect or no reason. As for question 2, 37.5% of the participants provided conceptual reasoning while 42.4% and 20.1% provided procedural and incorrect reasoning respectively. Finally, 33.1% of the participants chose conceptual reasoning while 40.8% and 26.1% chose procedural and incorrect reasoning for question 3. About only half of the participants were able to provide conceptual reasoning for question 1 which tests on inverse operations. The majority of the participants had provided procedural reasoning for question 2 and 3.

Table 2

Participants' types of reasoning for the three items

	Conceptual (%)	**Procedural (%)**	**Incorrect (%)**
Question 1	52.6	36.0	11.4
Question 2	37.5	42.4	20.1
Question 3	33.1	40.8	26.1

Table 3 depicts the total scores of the participants. The highest score for each question is 3. Some 33.1% of the participants managed to score full marks for question 3, which means, they are aware of inverse of operations and are able to provide conceptual understanding. As for question 2 and 3, only 19% and 8.9% of the participants were able to demonstrate conceptual reasoning and correct solution, respectively, this creates an awareness that the majority of participants are unable to understand the relationship between addition and subtraction. The nature of the item without actual numbers, reflects their inability to work with addition and subtraction when there is no number given. If a question such as $10 + 5 - 3$ is given, they would definitely be able to give the correct solution. However, when there is

no initial number given and they were required to think what happens if they add five first and then subtract 3 (see question 2 in Appendix), the participants' performance was extremely poor as shown in Table 3. Score of 2 and 1 shows the participants were able to provide at least correct answer or procedural reasoning, or both. Score of 0 depicts the participants were not able to perform at all in the both sections for an item.

Table 3

Total scores of participants

	Total Score			
	3	**2**	**1**	**0**
Question 1	33.1%	45.8%	12.9%	8.2%
Question 2	19.0%	36.1%	35.0%	9.9%
Question 3	8.9%	48.8%	31.7%	10.7%

Conclusion

This study sought to examine Year Five pupils' understanding of the relationship between addition and subtraction. Question 1 examined the participants' understanding on the inverse relationship, which means addition is the inverse of subtraction and vice versa. The initial number will remain the same when adding and subtracting a same number to it. This is a very fundamental concept of arithmetic. This also plays an important role in algebra. This concept is often used in formal algebra while solving an equation. Creating an awareness of this concept will enable primary pupils to understand the use of inverse relationship right from the beginning. Failure to grasp this concept might lead them to merely memorize the algorithm and apply it while doing formal algebra in later years of education. The results of this study exhibit that many are still unaware about this concept and failed to score in this item.

Question 1 and question 2 examined the participants' understanding on the relationship between addition and subtraction. These items were developed to test participants' understanding of direction of change while adding and subtracting. Adding a greater number than the number being subtracted should yield a greater number than the initial number and vice versa. This concept tests on properties of operations which focuses on mathematical processes. Getting a correct solution when performing addition or subtraction does not guarantee the students' conceptual understanding. In-depth understanding of the properties of operations will lead the students to see the connection between arithmetic and algebra.

The findings of this study showed the Year Five pupils' in a district of Malacca performed below average in the items involving properties of operations. There was no previous study made on investigating year five pupils' understanding on properties of operations. As such, this study could be an eye opener to researchers and educators in Malaysia to look into this aspect. Study conducted by Ralston (2013) using US and Singapore sample showed the understanding of age 11 pupils on properties of operations comparatively better. However, the Singapore pupils pose better understanding of properties of operations compared to US children. This creates an early awareness to the Malaysian mathematics curriculum developers on the algebraic thinking elements in primary schools. Future studies should be conducted using wider geographical areas in Malaysia and evaluate primary pupils' algebraic thinking skills. Malaysian mathematics curriculum developers should consider integrating algebraic thinking elements in the syllabus. Professional development for the teachers should be given in order to train them on how to infuse algebraic thinking elements while teaching arithmetic.

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APPENDIX

Question 1 :

Soalan 1:

Atika is thinking about a number. If she adds 5 to that number and then subtracts 5, what will happen to that number?

Atika memikir satu nombor. Jika dia menambah 5 kepada nombor tersebut dan kemudian menolak 5, apakah akan terjadi kepada nombor tersebut?

A. That number will increase.
Nombor itu akan bertambah.

B. That number will decrease.
Nombor itu akan berkurang.

C. No changes.
Tiada perubahan.

D. I can't tell without calculation.
Saya tidak dapat menentukannya tanpa pengiraan.

Reason:

Sebab:

A. It is because adding and subtracting the same number is like adding a zero to that number, so the result stays the same.
Ini kerana menambah dan menolak nombor yang sama adalah seperti menambah kosong kepada nombor tersebut, maka jawapan adalah tetap sama.

B. For example, if the number Atika is thinking is 2; when she adds 5 to 2 and then subtracts back 5; the number remains 2. Therefore the result stays the same.
Contohnya, jika nombor yang difikir oleh Atika adalah 2; apabila menambah 5 kepada 2 kemudian menolak 5; nombor itu kekal 2. Maka jawapan adalah tetap sama.

C. It depends on what number Atika is thinking.
Ia terpulang kepada nombor yang difikirkan oleh Atika.

Question 2 :

Soalan 2:

Siva is thinking about a number. If he adds 5 to that number and then subtracts 3, what will happen to that number?

Siva memikir satu nombor. Jika dia menambah 5 kepada nombor tersebut dan menolak 3, apakah yang akan terjadi kepada nombor tersebut?

A. That number will increase.
Nombor itu akan bertambah.

B. That number will decrease.
Nombor itu akan berkurang.

C. No changes.
Tiada perubahan.

D. I can't tell without calculation.
Saya tidak dapat menentukannya tanpa pengiraan.

Reason:

Sebab:

A. Let's say, if the number Siva is thinking is 2; when he adds 5 to 2 and then subtracts 3; the answer is 4 which is larger than 2. Therefore, he will end up with a larger number.
Katakan, nombor yang difikir oleh Siva adalah 2; apabila dia menambah 5 kepada 2 dan menolak 3; jawapannya adalah 4 yang mana lebih besar daripada 2. Maka, dia akan mendapat nombor yang lebih besar.

B. It depends on what number Siva is thinking.
Ini kerana ia terpulang kepada nombor yang difikirkan oleh Siva.

C. It is because Siva is adding more than what he subtracts. Therefore, he should end up with a larger number.
Ini kerana Siva menambah lebih daripada apa yang ditolak. Maka, dia sepatutnya mendapat nombor yang lebih besar.

Question 3 :

Soalan 3:

Mei Mei is thinking about a number. If she subtracts 7 from that number and then adds 5, what will happen to that number?

Mei Mei memikir satu nombor. Jika dia menolak 7 daripada nombor tersebut dan kemudian menambah 5, apakah yang akan terjadi kepada nombor tersebut?

A. That number will increase.
Nombor itu akan bertambah.

B. That number will decrease.
Nombor itu akan berkurang.

C. No changes.
Tiada perubahan.

D. I can't tell without calculation.
Saya tidak dapat menentukannya tanpa pengiraan.

Reason:

Sebab:

A. Let's say, if the number Mei Mei is thinking is 10; when she subtracts 7 from 10 and then adds 5; the answer is 8 which is smaller than 10. Therefore, she will end up with a smaller number.
Katakan, nombor yang difikir oleh Mei Mei adalah 10; apabila dia menolak 7 daripada 10 dan menambah 5; jawapannya adalah 8 yang mana lebih kecil daripada 10. Maka, dia akan mendapat nombor yang lebih kecil.

B. It is because Mei Mei subtracted a larger number than what she added.
Ini kerana Mei Mei menolak nombor yang lebih besar berbanding dengan nombor yang ditambah.

C. It is because Mei Mei performed addition lastly and addition will make numbers larger.
Ini kerana Mei Mei melakukan penambahan akhir sekali dan penambahan akan meningkatkan nilai sesuatu nombor.