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The Role of Representations in the Understanding of Mathematical Concepts in Higher Education: The case of Function for Economics Students

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Abstract: There are numerous studies about the teaching and learning of mathematics at different educational levels. In the case of higher education most studies were conducted at pedagogical departments for prospective teachers and mathematical departments. The present study concentrates on university students who attend a course on mathematics as part of a program at the Faculty of Economics and Management. It examines aspects of students' affective and cognitive behavior in solving representation tasks concerning their understanding of exponential and logarithmic functions. Results confirmed the existence of a comprehensive model with significant interrelations among general beliefs, self-efficacy beliefs and cognitive behaviour about the use of representations in general and, in the case of the specific concept. Regression analysis indicated the predominant role the self-efficacy beliefs play in the use of representations in defining the concept of function and solving recognition and translation tasks. Implications about the teaching of mathematics in higher education are discussed.

Keywords: *self-efficacy beliefs, function, general beliefs, representations*

Introduction

Mathematics plays an increasing role in many different scientific disciplines in general and in Business Administration and Economics Studies in particular. The teaching of Economics is based on the use of analytical models that require a high level of understanding mathematics (Barnett, 2009). As Vosskamp (2015) argues many empirical studies about economic issues have gained importance in recent years due to the availability of technical resources offered by Mathematics. A practical consideration that economics instructors should be aware of is that first-year economics courses run parallel to mathematics courses and students' mathematical knowledge depends on their mathematical background derived from their experiences in secondary education. Cai, Perry, Wond and Wang (2009) conducted interviews with teachers worldwide and they found out that there was an agreement between them that mathematics is applicable to real life problems and to other disciplines. The value of applicability of the mathematical concepts on the discipline of Economics is extremely important in higher education

Students at the beginning of their higher education cannot undoubtedly be considered tabula rasa. They have previous knowledge, experiences, beliefs, self-efficacy beliefs, motivations, cognitive and learning styles etc. Many different studies examined some of the personal factors which influence students' academic performance and the impact of the variables derived from the formal or informal teaching and learning environment. The present study interrelates the students' general beliefs and self-efficacy beliefs, as part of the affective domain with their inability to use many different representations for the same concepts in the learning of mathematics. It concentrates on a central concept in the case of the Economics, the concept of function.

Representations introduce a concept in different ways. For example, an exponential function can be represented in symbolic, numerical, graphical, and geometric forms. The use of those representations flexibly and fluently is positively correlated with the academic performance in mathematics (Gouya & Sereshti, 2006). The concentration on the concept of function is because function plays an important role in algebra and trigonometry which leads to the learning of calculus. "Understanding the concept of function from cognitive perspectives implies an ability to make connections between different representations of the concept" (Siti, 2010, p. 282). Pettersson (2012) argued that function is a threshold concept in mathematics: it is transformative as the understanding of the concept that leads to a new perception of the subject; it is integrative as the new understanding reveals connections with other topics and troublesome, as it presents difficulties to students.

There are many studies about the role of representations in teaching mathematics at different educational levels, starting from early years (e.g., Sterner, Wolff & Helenius, 2020) continuing with primary education (e.g., Panaoura et al., 2009), secondary education (e.g., Daryaee et al., 2018; Gagatsis et al.,2002; Gagatsis & Shiakalli, 2004) and, finally, at higher education mainly in pedagogical departments (e.g., Ballard, 2000) and mathematical departments (e.g. Moru, 2009; You & Quinn, 2010). Studies on higher education are fewer than other stages and our knowledge about the relations between teaching and learning and about the interrelations between internal and external learning factors is incomplete.

Everyone's actions when confronted with a task and mainly when he/she comes up against cognitive obstacles and difficulties, is determined by the respective aspects of the affective performance, such as attitudes, beliefs, values, motivations, self-efficacy beliefs. The present study concentrated only on two dimensions which seemed based on our previous studies (e.g., Gagatsis et al., 2017; Deliyianni et al., 2017; Panaoura et al., 2010) at different ages and concepts, to be related with the students' performance when they have to use different types of representatios: beliefs and self-efficacy beliefs.

Theoretical framework

The teaching of mathematics at higher education: Higher education must prepare graduates with necessary knowledge and skills (Ding et al., 2016) and provide them with the opportunity to tackle successfully the current and future professional challenges. Maron (2016) discussed the role of mathematics in Economics and identified that for many tasks in Economics, mathematical methods are used mechanically without taking into consideration the limitations of the methods. A usual misconception of many students entering higher education is that the relevant subject consists of a large amount of knowledge and a mastery of rules and processes (Fry et al., 2009). The results of an open-ended question addressed to university students of a mathematical department and concerning their best way mathematics should be taught, indicated that most of them (45%) preferred the use of examples, 23% the use of visualization (Borromeo-Ferri, 2015) and only 1.6% asked for formalizing steps.

According to Mardanov and Khasanova (2014) the future qualified economists need serious mathematical training that would enable them to use mathematics to study a wide range of economic problems. However, the students of the specific field of studies tend to have a weaker background in mathematics when entering university than they used to in past (Viirman, 2014). Possible reasons are the entry requirements (which in some cases do not include mathematics) or to the difficulties which are related with the teaching and learning processes in secondary education.

However, academic teaching is expected to tackle the difficulties and overcome the obstacles to fulfil the expected learning outcomes of each graduate scientist. University teaching academics are often expected to develop their understanding of teaching and learning on their own. That means that they must use the appropriate teaching methods to fit students with different backgrounds and inter-individual differences the way they think, study and learn. This expectation is clear for the academics of the pedagogical departments, who in many cases develop relevant in-service training programs for the academics of other departments (as for example, the programs at the capacity building centres for the improvement of teaching methods). However, “anyone teaching in higher education knows that it is not so easy to decide what works and what does not work when teaching in their discipline” (Rethiaume, 2009, p. 2015) and we have to present practical suggestions for the academics who teach mathematics in programs where it is not the main discipline.

The role representations play in mathematics and mainly in the understanding of the concept of function: Two conferences were held concerning the use of representations in the teaching of mathematics in 2009 and 2010 in Michigan, indicating the significance of the specific domain for the relevant international community. Mathematics deals with special objects and their properties which are studied via their representations (Dorfler, 2015). It is important for students to be encouraged to represent mathematical ideas in a variety of ways that make sense to them (Daryaee et al., 2018). Within this framework, when students study a new topic in mathematics, they confront a lot of new representations of the mathematical concepts. Therefore, they need to make appropriate relationships between those representations. Ballard (2000) underlined that the students need extensive practice to translate between representations and understand the different dimensions of each representation. A student translates a representation from one mode to another or he/she transforms a representation into another in the same mode (Kastberg, 2002).

The duality of internal vice versa external representations is a key notion for Goldin (2008). External representations comprise of the conventional symbol system of mathematics such as the formal algebraic notation, while the internal representations comprise students' personal meaning to mathematical concepts (Godino & Font, 2010). Given that a representation cannot fully reflect a mathematical construct and that each representation has different limitations, the use of various representations for the same mathematical situation is at the core of understanding. When students are asked to define a mathematical concept formally or in their own words and solve tasks related with the concept, they are expected to use external representations to express their internal representations.

Duval (2006) claimed that mathematical activities clearly involve changes from one representation to another. He focused on the mathematical discourse itself and argued that semiotic representations are the only way for us to get access to mathematical objects. The ability to convert from one representation to another and interact procedurally and conceptually with representations is a type of flexible mathematical thinking. Daryaee et al. (2018) argued that one of the ways of making the process of learning algebra meaningful is to use representations. In their study they showed that the use of different representations in teaching had a positive effect on learning algebraic concepts. Carson, Oehrtman and Engelke (2010) underlined the value of being able to work with different representations in the case of the concept of function. O'Callaghan (1998) suggested a conceptual model to describe the understanding of the concept of function with four main competencies: (a) modelling to represent a problem by using functions, (b) interpreting representations of functions, (c) translating from one representation to another and (d) creating a mental object from a process or procedure. Panaoura et al. (2015) investigated students' ability to understand the concept of function in secondary education by asking them to define it and solve tasks which asked them to flexibly manipulate the concept in different forms of representations.

According to Clement (2001) the typical mathematics definition of function from x to y is a correspondence that associates with each element of x a unique element of y. Students' concept image usually differs greatly from a mathematical acceptable definition. For example, it may be limited to a graph of a relation that passes the vertical – line test and students usually believe that a function must include some algebraic formula. At their study they asked students to provide their own definition of a function. Only 10% of precalculus students could give a definition which was like the mathematical definition of function. They concluded that, although the concept of function is central to understanding mathematics, students' understanding of functions appears to include erroneous assumptions.

There are numerous studies in students' conceptions of the function concept showing inconsistencies between conceptions and definitions (Viirman, 2014), as the structural nature of the set-theoretical definition is problematic for learners. Kastberg (2002) suggested that even though students were supposed to understand the concept of logarithmic function, they could not remember or use its properties in subsequent courses in higher education. Students' difficulties, understanding and misunderstandings need to be examined and interpreted in the context of interrelated factors.

Comprehensive model of cognitive and affective performance: Someone's cognitive behaviour when confronted with a task, is determined by the respective beliefs, self-efficacy beliefs and personal theories rather than the knowledge of the task. Affective factors, such as self-efficacy beliefs, motivation, engagement, attitudes, and beliefs towards mathematics play a major role in success or failure of mathematics learning (Abdulwahed et al., 2012). "A persistent problem in understanding the role of affect in mathematics teaching and learning has been to settle on a clear definition of what is affect" (Ignacio et al., 2006, p. 16). Most researchers accept emotions, attitudes, beliefs, and values as the key components of the affective domain in mathematics education and use them to study the interactions among cognition, problem solving, teaching, and learning processes and achievements (Beltran – Pellicer & Godino, 2019).

Students at higher education have various beliefs concerning mathematics and its learning: about themselves as learners of mathematics, the nature of mathematics, the way in which the knowledge is acquired, the factors that affect the learning of mathematics, the impact of their previous school experiences. A part of those beliefs constitutes their self-efficacy beliefs which were mainly defined by Bandura's social cognitive theory (Bandura, 1997), in order to explain one's perceived ability to plan and execute specific tasks.

Berthiaume (2009), based on the knowledge about teaching and research in disciplinary specificity in university teaching suggested a model of discipline – specific pedagogical knowledge. The strong interrelations among all the related factors are its main characteristic: (a) the factor about the knowledge base of teaching consists of the goals, the knowledge and the beliefs related to teaching, (b) the factor about the disciplinary specificity is consisted of the epistemological structure and the socio-cultural characteristics and (c) the factor about the beliefs consists of the beliefs about knowledge and knowing, knowledge construction and the evaluation of knowledge. Teachers' beliefs and practices influence undoubtedly students' beliefs, self-efficacy beliefs, conceptions, and abilities. A teacher's manner of presenting mathematics influences the ways that students understand and learn mathematics. If we present mathematics as a logical system of thought we may create the perception of mathematics as a collection of facts that have to be memorized (Kyle & Kahn, 2009). On the contrary, if students are activated creatively in mathematics they will focus on the use of different strategies and realize the value of mathematical ideas and problems. In the case of the concept of function, according to a standard didactic sequence in secondary education, students are given a graph of a function and they must infer from it some properties of the function by following a specific procedure (Sajka, 2003).

Research in the relation between the affective domain and mathematics focuses usually on the relation between students' attitudes or beliefs and mathematical performance. It also seems necessary to consider the epistemic component (Beltran – Pellicer & Godino, 2019) about the nature and the value of concepts. Bosse et al. (2011) discuss teachers' beliefs and the respective instructional practices concerning the use of representations and the mathematical translations. They concentrated on the translations which students find difficult and suggested that teachers should come to understand their own beliefs regarding the different translations. At the same time, they must guide students to self-evaluate and self-manage their beliefs and abilities of using different tools and different representations in order to solve mathematical tasks on specific concepts. A similar and at the same time different dimension is students' beliefs and self-efficacy beliefs concerning the nature, the value, the processes, and the strategies. Those variables are related with the understanding of any mathematical concept.

The present study: In 2015 a conference was organized in Germany about the "Didactics of Mathematics in higher education as a scientific disciplinary". The 3rd one of the nine working strands was about Mathematics as a service subject in Engineering and Economics, while the 5th one was about students' motivation, beliefs, and learning strategies. Functions are central to present day mathematics, and they are widely used in the comparison of abstract mathematical structures (O'Shea et al., 2016). They also seem to be a central concept in Economics.

The aim of the present study was twofold: (a) to examine the students' ability to solve fluently and flexibly tasks concerning the exponential and logarithmic functions in relation to the use of the respective representations, (b) to confirm a comprehensive model and examine the interrelations between a part of the cognitive (solving tasks) and affective factors (general and self-efficacy beliefs) concerning the use of representations in general and the understanding of the concept of function in particular.

The originality of this study lies in the fact that it uses data from university students, while previous similar studies in the use of representations of the concept of function concentrated on students' performance in Mathematics at secondary education (Panaoura et al., 2017; Elia et al., 2007). It is important to examine and discuss the teaching of mathematics in higher education through an interdisciplinary perspective, as a tool for the development and understanding of concepts useful for other sciences. The present study concentrates on students who attend a course in mathematics in order to be able to use them in their major studies which were related to Economics and Management. One of the problems of the teaching of mathematics in courses in which mathematics is not the students' major is the need to assign specific meaning to most mathematical concepts (Mardanov & Khasanova, 2014). Students of the Faculty of Economics and Management are expected to use exponential and logarithmic functions to construct and interpret economic models. The concept of function admits a variety of representations. They are expected to use fluently and flexibly different representations to understand the concept of function. Which are their beliefs concerning the use of different representations of the same concept? Which are their respective self-efficacy beliefs, and which is their impact on their ability to understand the concept?

Theoretically the findings of this study will make a significant contribution in our understanding of the interrelations of beliefs and self-efficacy beliefs constructed in secondary education and are revealed in higher education, about the nature and value of mathematics in general and the use of representations with the students' ability to solve relevant tasks. A confirmation of a comprehensive model improves our knowledge of the complicated processes of learning and understanding. Practically, such an investigation should provide information about what can be emphasized by the teaching of mathematics in higher education based on dimensions of students' cognitive and affective behavior.

Methods

There are two main different approaches to the mathematics education of students of Business Administration and Economics study programs: a) mathematics courses are offered by Mathematical departments and b) the courses are organized by the Business Administration and Economics department. In the case of the University of Cyprus, where the specific study was conducted, the first approach is used. The University of Cyprus is the only public university which offers the specific programs of studies. As a public university, students' entrance requirement depends only on the competitive entrance exams organized by the Ministry of Education, Culture, Athletes and Youth. All students are obliged to choose Mathematics as one of the main four subjects to be examined in the entrance exams. However, there is not any mark which needs to be achieved to get a place. For example, a student can have gained the 80th position in the faculty by having a total mean score of 17/20, although he/she may have 9/20 in mathematics.

Participants

The study was conducted among the undergraduate students (371 students) who study in the Faculty of Economics and Business Management in three different departments at the beginning of the second semester (they did not attend any mathematics course during the first semester): the department of Economics, Accounting and Finance, Business and Public Administration at the public university in Cyprus.

Procedure and research tools

A questionnaire was used to measure students' beliefs and self-efficacy beliefs. A test was also used in order to measure their ability to solve tasks involving different representations. Both the questionnaire and the test were constructed by the authors of the paper.

The questionnaire: A questionnaire was compiled to investigate the students' beliefs and self-efficacy beliefs concerning Mathematics, the use of representations in general and exponential and logarithmic functions. The questionnaire consisted of 56 Likert-type items of five points (1=strongly disagree, 5= strongly agree). The reliability of the questionnaire was high (Cronbach's alpha = 0.93). The items were content and face-validated by a Professor of Economics, an Associate Professor of Management and a Professor and an Associate Professor of Mathematics Education. The questionnaire consisted of statements, which were divided into two main categories. The first category investigated students' beliefs concerning mathematics which included the following dimensions: (a) beliefs about the value of Mathematics as a tool in relation to their studies, (b) beliefs about the value of using different representations in general, (c) beliefs about the value of using representations in relation to their studies, (d) beliefs about the value of using representations to solve tasks of exponential and logarithmic functions in relation to their studies. The second category investigated students' self-efficacy beliefs about: (a) their Mathematics achievement, (b) the use of representations and (c) the use of representations of exponential and logarithmic functions. The questionnaire was administered to the students by the researcher who is one of the authors under usual classroom conditions after explaining to them the aim and the significance of the present study. Lecturers left the room, providing about 40 minutes at the end of the teaching period. Indicative items of each category are presented:

Beliefs about Mathematics (BM)

- Mathematics is essential in my work.
- Economic analyses which are emphasized in mathematical models correspond to reality.

Beliefs about the use of representations in mathematics (BR)

- The graphical representation is an important way of understanding a concept.
- The exercises in which graphical representation is given are easier to solve.

Self-efficacy beliefs about mathematics (SEBM)

- I am able to understand mathematical concepts and procedures.
- I have the necessary skills to use and apply Mathematics

Self-efficacy beliefs about the use of representations (SEBR)

- I am able to convert one representational system into another (e.g., from algebraic to graphical)
- I am able to apply properties to process a representation.

The test: A test was compiled to investigate the students' achievement concerning the use of representations in exponential and logarithmic functions. The test comprised ten tasks/exercises, divided into four different categories. The first category investigates the definition of the concept of function, the second examines the translation from one type of representation to the other, the third investigates the recognition abilities and the fourth category concerns problem-solving abilities. The problem-solving ability is not examined in the present study, as we concentrate our attention on the definition of the concept and the solving of tasks by handling different representations. The tasks which were used for the analyses in the specific study are presented at the Appendix and they are divided into three main categories:

(a) Definition tasks (Q1DEF, Q1DEFexp, Q3a, Q3b and Q8),

(b) Recognition tasks (Q4, Q4exp, Q9 and Q9exp)

(c) Translation tasks (Q2, Q2exp, Q5, Q5exp, Q6, Q6exp, Q7 and Q7exp)

Statistical Analysis

To examine students' performance in understanding the concept of function and using the different representations fluently and flexibly in relation to their respective beliefs and self-efficacy beliefs, descriptive statistical methods, comparison of means and regression analysis were used. By having the factor "beliefs" as central, the sample was clustered into categories according to their beliefs and self-efficacy beliefs to examine any differences of their ability to define the concept and solve recognition and translation tasks. Additionally, we examined the impact of each affective factor the specific study measured on the students' performance in understanding the concept of function.

Finally, to confirm the structure of students' beliefs and self-efficacy beliefs in relation to their performance, according to the second research question, a confirmatory factor analysis model was constructed using Bentler's (1995) Structural Equation Modelling (EQS) programme. The tenability of a model can be determined using the following measures of goodness of fit: x2/degrees of freedom (df) < 1.95, (Comparative Fit Index) CFI >0.9 and (Root Mean Square Error of Approximation) RMSEA < 0.06. Firstly, confirmatory factor analysis was used in order to confirm the structure of the factors of beliefs and self-efficacy beliefs and then it was used in order to examine the interrelations among those affective variables with the performance in mathematical tasks.

Results

Variables of the test were divided into three main categories to have an indication of students' abilities to do: a) tasks about definition (Q1DEF, Q1DEFex-example, Q3a, Q3b-example, Q8), b) recognition tasks (Q4, Q4exp – explanation, Q9, Q9exp – explanation) and c) translation tasks (Q2, Q2exp – explanation, Q5, Q5exp – explanation, Q6, Q6exp- explanation, Q7, Q7exp – explanation). Table 1 presents the percentages of correct answers for each task and the general students' mean performance in the definition, recognition, and translation tasks.

The results concerning the definition tasks were disappointing, especially in respect to the first task (Q1DEF), which asked them to define the concept of function (20.7%) and explain how they understand that a graphical representation does not represent a function (Q3a, 27.3%). A further analysis of the few correct answers given for the task Q3a indicated that almost all of them explained it verbally ($\bar{X}$=0.89). However, students were able to present an example of a function correctly (97.7%) and an example of a relationship that does not represent a function (Q3b, 82.6%). The second category was consisted of two tasks concerning the recognition of functions (Q4 and Q9). In Q4, 42.4% of the students correctly recognized the graph that corresponds to the type of the logarithmic function and in Q9 49.3% of the students chose the correct graph of the equation of the logarithmic function. However, most of them were not able to explain the way they were thinking of in order to solve the tasks (only 27% and 29.5% were able).

Table 1

Students' Performance at Function Tasks

Category	Tasks	Percent	Mean	SD
Definition tasks	Q1DEF	20.7	0.57	0.25
	Q1DEFex	97.7		
	Q3a	27.3		
	Q3b	82.6		
	Q8	41.7		
Recognition tasks	Q4	42.4	0.41	0.39
	Q4exp	27.0		
	Q9	49.3		
	Q9exp	29.5		
Translation tasks	Q2	57.6	0.54	0.38
	Q2exp	59.1		
	Q5	53.8		
	Q5exp	21.3		
	Q6	44.3		
	Q6exp	53.4		
	Q7	37.1		
	Q7exp	35.8		

Results were higher in the case of the translation tasks than the recognition tasks. In the case of Q2, approximately 60% of the students had correctly matched the graph with the type of the equation of the function it expressed (57.6%). Similarly, in Q5 and Q6 tasks, where students were asked to match the equation of the function with the corresponding graphical representation, results indicated that 53.8% responded correctly to Q5 and 53.4% in Q6. The percentage of success was lower in the case of Q7 (37.1%). In all translation tasks students had difficulties in explaining how they worked to reach the solutions, in Q5 and Q7 (Q2exp: 59.1%, Q5exp: 21.3%, Q6exp: 53.4% and Q7exp: 35.8%).

The first research question examined students' performance in relation to their beliefs and self-efficacy beliefs. Based on the content analysis of the items of the questionnaire we concentrated our attention only on the variables which constituted the beliefs concerning the nature and value of Mathematics (BM), the beliefs and the value of using different representations (BR), self-efficacy beliefs concerning their mathematical performance (SEBM) and self-efficacy beliefs concerning the use of representations in general (SEBR) and in the case of exponential and logarithmic functions in particular (SEBRexp and SEBRlog). The means and the standard deviations of students' responses are presented on Table 2. They had positive beliefs and self-efficacy beliefs concerning all the dimensions which were examined.

Table 2

Means and Standard Deviations in Students' Beliefs and Self-efficacy Beliefs

	$\bar{X}$	SD
BM	3.55	0.39
BR	3.83	0.50
SEBM	3.50	0.69
SEBR	3.31	0.37
SEBRexp	3.30	0.56
SEBRlog	3.22	0.44

The sample was divided into three groups, using cluster analysis in respect to their beliefs about mathematics (BM) in order to examine whether there were any differences in their performance in the definition, recognition and translation tasks. There were not any statistically significant differences (p=0.074). Similarly, the sample was divided into three groups in respect to their beliefs concerning the use of representations in mathematics (BR). 75 of them constituted the group with the lower beliefs ($\bar{X}$=3.13), 160 of them constituted the second group ($\bar{X}$=3.76) and 117 students had the highest beliefs ($\bar{X}$=4.38). In all cases there were statistically significant differences (p<0.01). Table 3 presents the results of the Anova mean comparison of the three groups in the definition, recognition, and translation tasks in relation to their beliefs concerning the use of representations.

Table 3

Students' Performance in Relation to their Beliefs Concerning the use of Representation

Ability	F	Mean
Definition	$F_{2,263}$=52.398, p<0.01	$\bar{X}1$=0.36 $\bar{X}2$=0.50 $\bar{X}3$=0.72
Recognition	$F_{2,246}$=64.170, p<0.01	$\bar{X}1$=0.08 $\bar{X}2$=0.28 $\bar{X}3$=0.68
Translation	$F_{2,61}$=17.183, p<0.01	$\bar{X}1$=0.13 $\bar{X}2$=0.20 $\bar{X}3$=0.63

Notes. 1= low beliefs, 2= medium beliefs, 3= high beliefs

It is obvious in all cases that students with high beliefs concerning the use of representations had statistically significantly different performance in definition, recognition, and translation tasks from the group of students with low and medium beliefs. Those differences were more apparent in the case of the recognition and the translation tasks.

Similarly, the students were divided into three groups, using cluster analysis, based on their self-efficacy beliefs concerning the use of representations in mathematics learning (SEBR). In the first group there were 37 students with low self-efficacy beliefs ($\bar{X}$=2.62), in the second group there were 199 students with medium self-efficacy beliefs ($\bar{X}$=3.23). Finally, in the third group there were 106 students with high self-efficacy beliefs ($\bar{X}$=3.71). Table 4 presents the results of the Anova mean comparison of the self-efficacy beliefs concerning the use of representations in mathematics, at the definition, recognition tasks and translation tasks.

Table 4 - caps

Students' performance in relation to their self-efficacy beliefs concerning the use of representations

Ability	F	Mean
Definition	$F_{2,259}$=68.128, p<0.01	$\bar{X}1$=0.30 $\bar{X}2$=0.48 $\bar{X}3$=0.75
Recognition	$F_{2,237=}$62.585, p<0.01	$\bar{X}1$=0.00 $\bar{X}2$=0.27 $\bar{X}3$=0.69
Translation	$F_{2,62}$=18.523, p<0.01	$\bar{X}1$=0.04 $\bar{X}2$=0.47 $\bar{X}3$=0.88

*Notes.*1= low beliefs, 2= medium beliefs, 3= high beliefs

As it was expected, students' performance in the three categories of tasks (definition, recognition, and translation) was related directly with students' self-efficacy beliefs concerning the use of representations in mathematics. Results on Table 4 indicated that there were in all cases statistically significant differences (p<0.01) with the highest performance presented by the students with high self-efficacy beliefs.

Finally, regression analysis was conducted in order to examine the impact of students' beliefs concerning mathematics (BM), the use of representations (BR), self-efficacy beliefs concerning mathematics (SEBM) and self-efficacy beliefs regarding the use of representations (SEBR) on their ability to define the concept of function to solve recognition and translation tasks. Results are presented on Table 5.

Table 5 - Caps

The impact of affective factors on students' mathematical performance

Ability	Equation	R^2	Beta
Definition	0.09BM+0.13BR+0.103SEBM+0.213SEBR	0.54	BM 0.143 BR 0.256 SEBM 0.272 SEBR 0.303
Recognition	0.14BM+0.246BR+0.163SEBM+0.387SEBR	0.64	BM 0.139 BR 0.307 SEBM 0.272 SEBR 0.335
Translation	0.073BM+0.253BR+0.114SEBM+0.473SEBR	0.65	BM 0.076 BR 0.334 SEBM 0.194 SEBR 0.441

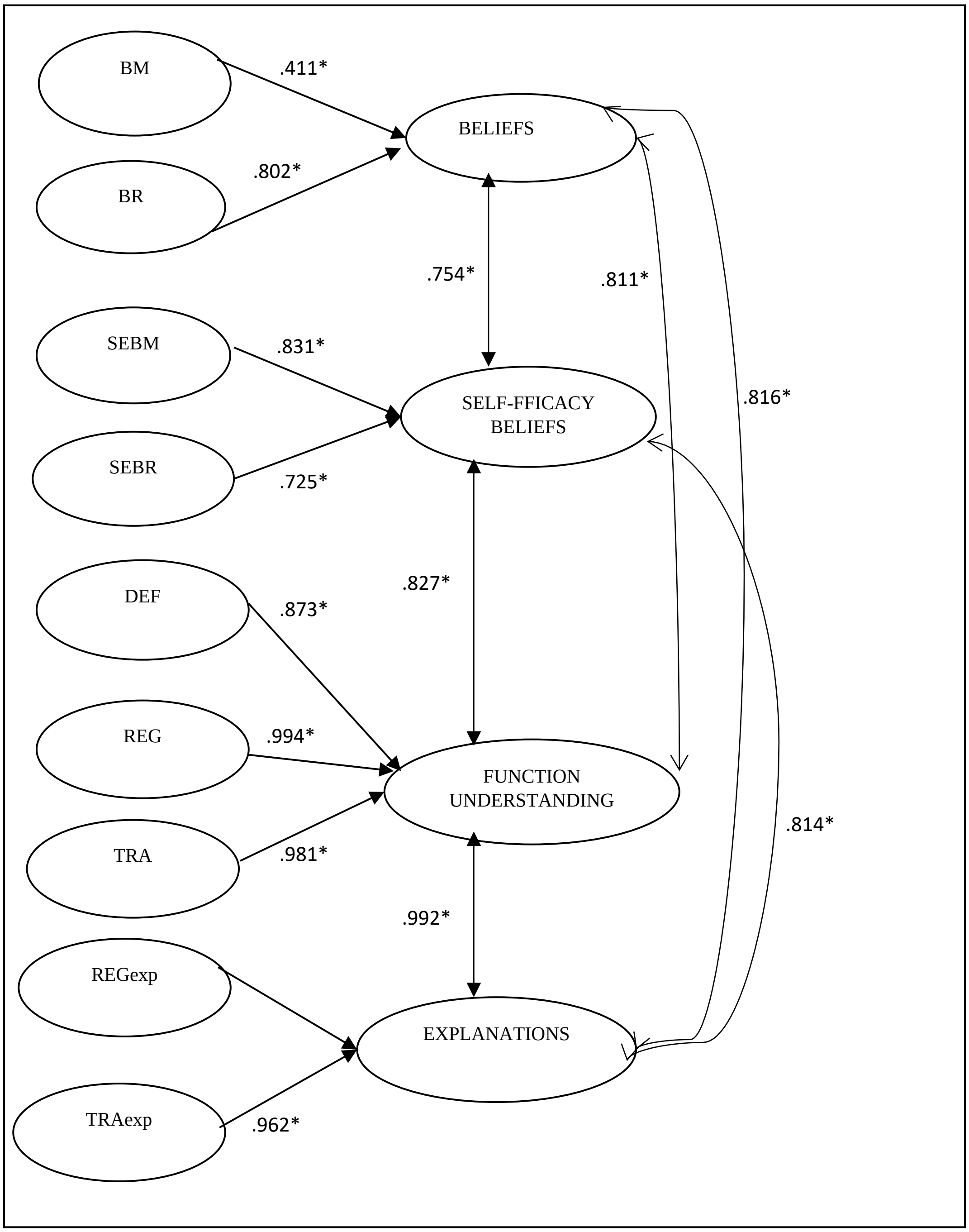
In all cases the results revealed the significant and predominant role of the self-efficacy beliefs concerning the use of representations on the students' performance. In the case of the recognition and translation tasks it had the highest impact (0.335 and 0.441 respectively) on the performance, while the second highest impact derived from their beliefs concerning the use of representations. In the case of the definition tasks the impact of self-efficacy beliefs concerning mathematics was higher than their beliefs concerning the use of representations. In all cases the beliefs concerning the nature and value of mathematics in general had very low impact on students' performance.

The second research question examined the interrelations between the cognitive (solving tasks) and the affective factors (beliefs and self-efficacy beliefs) concerning the use of representations in general and the understanding of the concept of function. The hypothesized model consisted of four first order factors: (a) students' beliefs, (b) their self-efficacy beliefs, (c) their performance in understanding the concept of function and (d) their ability to explain the way they worked on function tasks. The statistically significant interrelations between those factors were mainly examined.

According to Figure 1, the students' "beliefs" consisted of their beliefs concerning the nature and value of mathematics (BM) and their beliefs concerning the value of using different representations in mathematics (BR). Similarly, the students' "self-efficacy beliefs" consisted of their self-efficacy beliefs concerning their performance in mathematics (SEBM) and the respective self-efficacy beliefs concerning their ability to use representations fluently and flexibly (SEBR). The students' performance on understanding the concept of function consisted of their mean performance derived from the tasks which asked them to define the concept (DEF), recognize the concept in different forms of representations (REG) and translate the concept in different forms of representations (TRA). Finally, the fourth factor was about their ability to explain the way they worked when solving recognition tasks (REGexp) and translation tasks (TRAexp). All the loadings of the variables on the first order factors were high (above 0.700) except the loading of the general beliefs concerning the nature and the value of mathematics in their general mathematical beliefs (0.411). The fit of the model was excellent, based on the accepted indices ($x2 = 21.95$, $df=18$, $x2/df=1.21$, $p<0.05$, $CFI=0.989$, and $RMSEA=0.052$). The most important finding was the extremely high interrelations between the factors which were in all cases statistically significant. However, there were differences in the loadings of those relations. The strongest relation was found to be between the students' ability to solve the function tasks with their ability to explain their way of thinking (0.992). Students who were able to explain how they reached a solution were able to solve the tasks correctly. The ability to explain something presupposes the conscious realization of the thinking steps, and to self-reflect on those steps. It is a higher order thinking process with a metacognitive perspective. The loadings of self-efficacy beliefs (0.827) and beliefs (0.711) in students' ability to understand the concept of function were high and it seems that there were factors which explained the inter-individual differences which were found, concerning students' performance.

Figure 1- Caps for title

The CFA model of students' beliefs, self-efficacy beliefs, function understanding and explanations



Discussion

The present study concentrated on the students' understanding of the concept of function in relation to two aspects of the affective domain, their beliefs and self-efficacy beliefs concerning mathematics in general and the use of representations in particular. There are many studies on the teaching of the concept of function in secondary education and the use of representations in primary and secondary education. In the case of higher education, the emphasis is on the teaching of mathematics to students in pedagogical and mathematical faculties. The present study concentrated on teaching mathematics to students who do not have mathematics as a major course in their studies, who need mathematical concepts to understand and construct concepts related to their basic studies. Economists use increasingly the mathematical methods to interpret and analyse the possible outcomes of economic activities. The concept of function is important to fulfil the goals of the analysis and interpretation of the economic models.

The results of the present study revealed the students' difficulties in understanding the concept of function at the level of higher education. Their main difficulties concentrated on defining the concept. In mathematics, the definition which is presented either by using a formal structure or through an intuitive perspective plays a major role in the construction of mathematical thinking. The present results confirmed the existence of difficulties in defining the concept of function which were previously identified in secondary education (Elia et al., 2007). The students' concept image in the case of function differs from the mathematical acceptable definition (Clement, 2001). It seems to be easier for them to present an example to explain a mathematical concept rather than present a definition, which they believe it must be verbal. Alkhateeb (2019) analysed the use of representations in textbooks and found that the use of verbal representations was high although their presence in the mathematics textbooks was low. At the same time in the case of exercise tasks the use of algebraic symbols is preferred, as mathematics instructors traditionally like to focus their instruction on the use of algebraic representations (Panaoura at al., 2009b). Almost all students were able to present an example of a function, with lower performance in presenting a counter example. We believe that the specific result is based probably on the relevant teaching processes which are used by teachers in secondary education during the teaching of the specific concept. On the other hand, results indicated that students' tendency to define the concept verbally rather than to use any other type or representation depends probably on a belief that the intuitive and informal presentation of their conceptions cannot be a part of the mathematical learning. Students had many difficulties in explaining the way they worked on the tasks. Even in the cases their solution was correct they did not present explanations. Probably they are not able to present them in a written way or they prefer to do it orally. The ability to present the way of thinking after completing a task, implies that they are able to self-reflect on the cognitive processes, by activating higher order metacognitive processes.

Students' performance was higher in solving tasks which were related to the use of representations rather than the definition tasks. Their results were higher in the case of the translation tasks than the recognition tasks. A future study could make theoretical suggestions and examine levels of understanding the concept of function based on students' performance in different type of tasks.

The mean comparison indicated that students' differentiated performance in understanding the concept of function was based on students' beliefs concerning the use of representations and their respective self-efficacy beliefs. The regression analysis confirmed the findings of the mean comparisons by indicating that in all cases the less impact derived from their general beliefs concerning mathematics. Many studies underline the role beliefs concerning mathematics plays in students' performance. The present study indicated the predominant role and impact of their specific beliefs concerning the use of representations and mainly their self-efficacy beliefs concerning the use of the representations. The anticipated improvement of students' performance is based on the existence of a reciprocal relationship between self-concept beilefs and academic achievements (Marsh et al., 2005). Students learn by connecting new ideas to prior knowledge (Siti, 2010). In the case of the specific concept, it depends on previous experiences in secondary education. Those experiences constructed at the same time their knowledge, their beliefs and their self-efficacy beliefs. Many students start their studies in higher education with many difficulties in abstract thinking, so the teaching of mathematics needs to balance the use of many different forms of representations. Emphasis should be given to modify the teaching of functions by involving both the content of the concept and the methodological approaches of teaching it.

The relationships between beliefs and self-efficacy beliefs with the learning of mathematics is not simple, linear or unidirectional (Grootenboer & Hemmings, 2007). Confirmatory factor analysis indicated that there is a coherent model of self-dimensions about the use of representations to understand the concept of function and the relevant mathematical performance. The significant interrelations underline the impact of dimensions of the cognitive domain on the affective domain and vice – versa (Schreiber, 2002), and the possible influence of a dimension on other dimensions of the same concept. Results confirm that students with lower performance have at the same time negative beliefs and self-efficacy beliefs concerning the use of representations as they are not able to use them fluently and flexibly. In the future, it would be interesting to examine the accountability of relevant intervention programs aiming to develop students' ability to understand the concept of function by improving the dimension of the affective domain and vice versa.

At school mathematics students get the feeling that mathematics describes situations like physics and economics (Dorfler, 2015). However, in higher education it seems that those beliefs concerning the value and significance of mathematics do not have a direct impact on their ability to understand the concept of function and use the different forms of representations fluently and flexibly. Working with functions in various contexts requires the ability to think flexibly as far as the concept of function is concerned and appreciate it as a "mathematical object" (O'Shea et al., 2016, p. 296). The general statements about the value of mathematics and its significant contribution to everyday activities which are probably useful in the ages of primary and early secondary education, are not convincing in higher education. Students at this level are interested in the significance of the knowledge in the context mainly of their studies and the future use in their professional activities. This is in line with Petocz et al. (2007) who indicated that an important dimension of curricula in higher education is making explicit connections between students' courses and the world of professional work. Thus, mathematics instructors should design learning tasks that reflect the way mathematics is used in their future professions.

Conclusion

The present study revealed the necessity to rethink of many issues concerning the teaching of mathematics in higher education. It seems that it is important to investigate how students use and react t each lerning tool, such as the use of representations and what beliefs and self-efficacy beliefs develop. Beliefs reveal the understanding and propositions about the world, while the self-efficacy beliefs indicate one's perceived ability to execute a task. Those self-concepts may fall at secondary education duew to uncertainties resulting from less personalized instruction and perceptions of increased academic processes. There is not a "teaching recipe" suitable for all the students, all the faculties, all the courses of higher education. Additionally, a model of mathematics education that is adequate today under specific circumstances may be inadequate in the future and in a different context. We must continue investigating the students' performance in higher education in relation to the related impact factors and the teaching processes in traditional teaching. We also must take into consideration that higher education mathematics instructors are probably excellent mathematicians with little pedagogical background. The suggestions for any teaching processes must be based on research findings in order to lead to more accountable results and be more convincing to the instructors. A future study could examine aspects of the students' affective and cognitive performance after attending specific mathematical courses during their studies in higher education.

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Appendix

Definition tasks

Q1DEF

What do we call "function"?

Q1DEFex

Give an example of function.

Q3a

How do we understand that a graphical representation (of an orthogonal axle system) does not correspond to a function?

Q3b

Give an example of a relationship that does not represent a function.

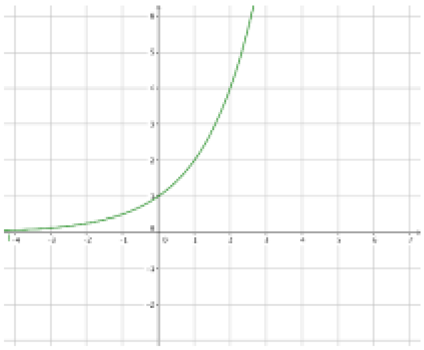
Q8

If f (-4) = 2 and f (-4) = 0, check if the relation f can be a function.

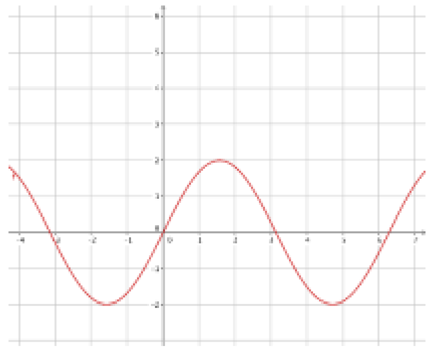
Recognition tasks

Q4

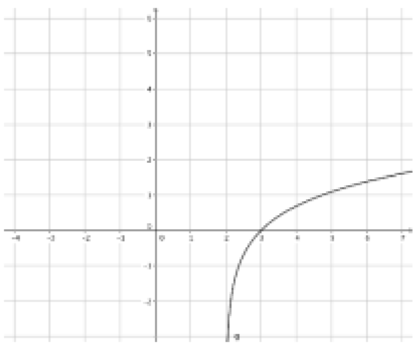
Which of the following graphs corresponds to a logarithmic function? (Choose the right one)



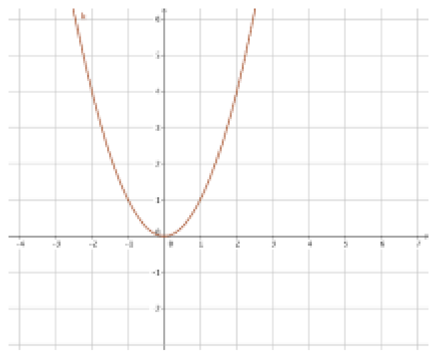
A



B



C



D

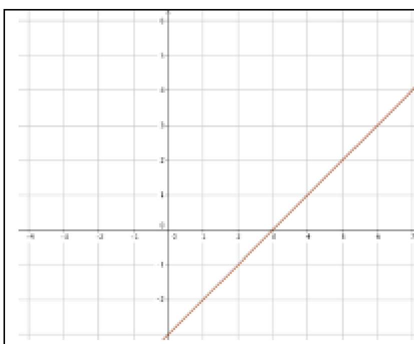
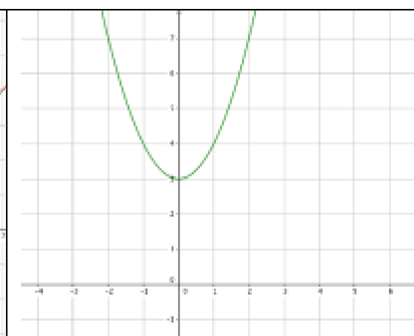
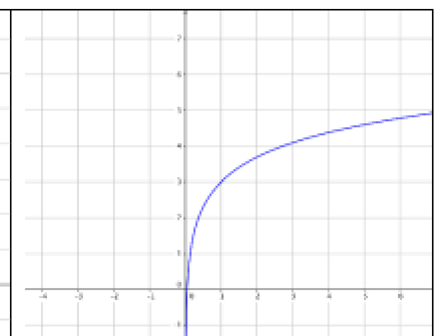
Q4exp

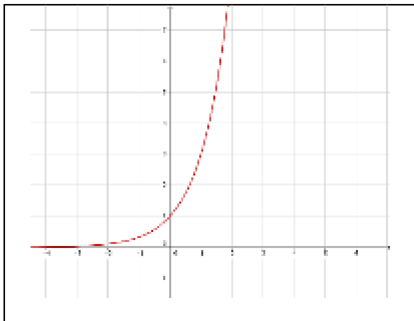
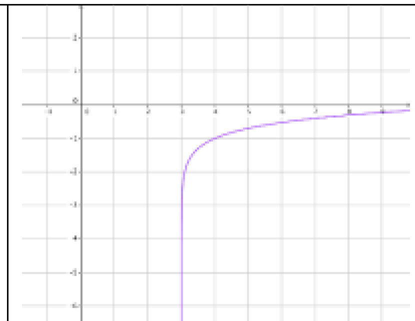
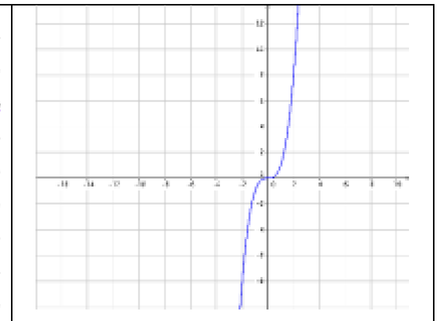
Explain the way you worked to solve the task.

Q9

Which of the following graphs could be the graph of equation f (x) = lnx + 3?

Choose the right one.

		
A	B	C

		
D	E	F

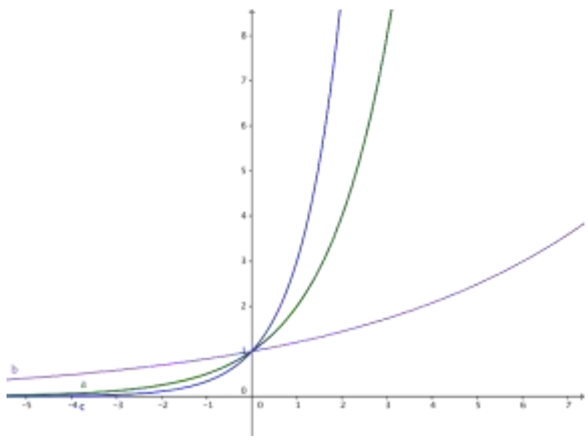
Q9exp

Explain the way you worked to solve the task.

Translation tasks

Q2

The figure below shows graphical representations of $f(x) = 2^x$, $g(x) = (1{,}2)^x$, $h(x) = 3^x$. Match each graph (a, b, c) with the appropriate equation.



Q2exp

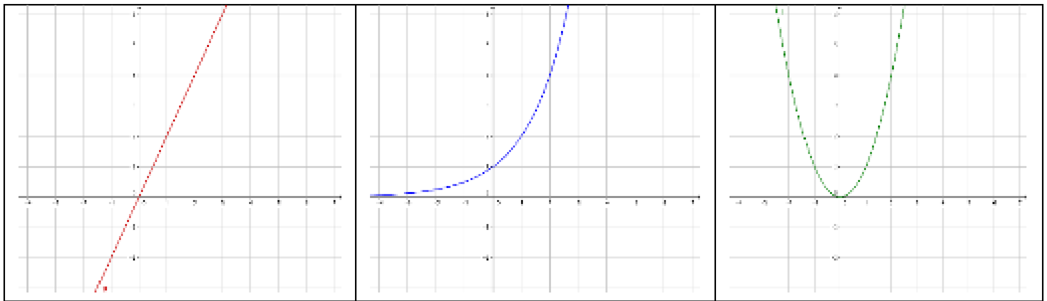
Explain the way you worked to solve the task

Q5

The following figure gives the graphs of the equations:

A. $f(x) = x^2$
B. $g(x) = 2x$
C. $h(x) = 2^x$

Write the appropriate equation under the correct graph



a.	b.	c.

Q5exp

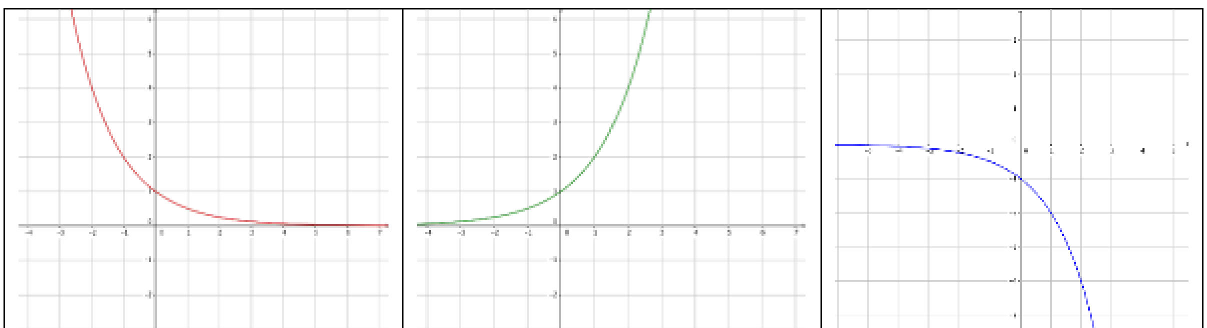
Explain how you worked

Q6

The following figure gives the graphs of the equations:

A. $f(x) = 2^x$
B. $g(x) = 2^{-x}$
C. $h(x) = -2^x$

Write the appropriate equation under the correct graph



a.	b.	c.

Q6exp

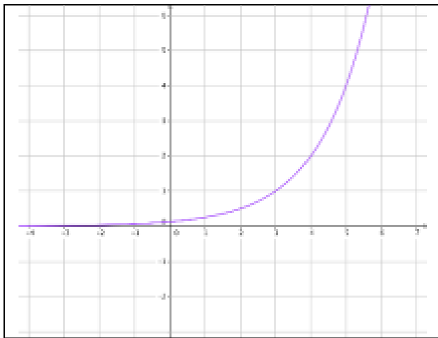
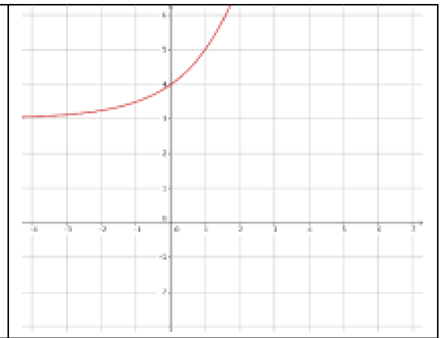
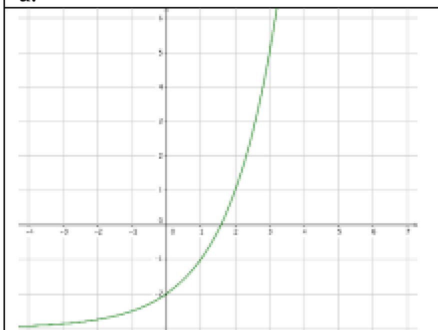
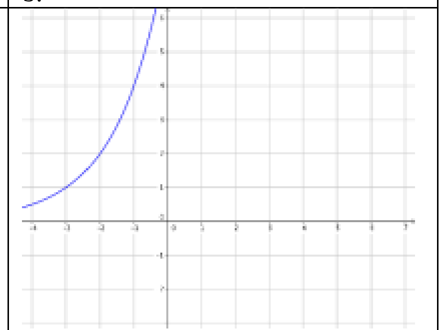
Explain how you worked.

Q7

The following figure gives the graphs of the equations:

A. $f(x) = 2^x - 3$
B. $g(x) = 2^x + 3$
C. $h(x) = 2^{x+3}$
D. $j(x) = 2^{x-3}$

Write the appropriate equation under the correct graph

	
a.	b.
	
c.	d.

Q7exp

Explain how you worked.

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