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Sociocultural influences on primary teachers implementing *TouchTimes*

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Abstract: In this article, I share case studies of two primary school teachers (K–5) in British Columbia, Canada who were interviewed after their implementation of *TouchTimes* (hereafter, TT) in their mathematics classes. TT is a multi-touch digital application that is designed for users to create and interact with multiplicative situations kinaesthetically through their fingertips on an iPad screen. Using the theoretical constructs of double instrumental genesis, instrumental distance and didactical landmarks, I identify and highlight sociocultural influences described by each of the teachers as being impactful on their integration of this digital technology into their mathematics teaching. These influences included other teachers and the researchers who were part of a larger research project involving TT, as well as the students in each of the case study teachers' classes. My analysis indicates the multi-faceted and complex nature of the process of double instrumental genesis that teachers undergo when implementing digital technology and how sociocultural factors impact teachers' personal and professional instrumental geneses.

Keywords: double instrumental genesis; elementary school teachers; multiplication; touchscreen technology; *TouchTimes*.
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Introduction

Although various forms of digital technology have been implemented in British Columbia schools since the early 1980s, it was the 2010 release of Apple's iPad, with its touchscreen interface, that has provided an alternative to the keyboard and mouse input mechanisms. This is particularly relevant for students in Kindergarten to grade three, whose fine motor skills are still developing and, therefore, have less dexterity for manoeuvring a mouse or using a keyboard efficiently. Touchscreen devices are more intuitive and user-friendly, and, through simple tactile actions such as pressing, tapping, swiping and pinching, users can interact with the technology directly through screen contact with their fingertips. Their ease of use, alongside the increasing availability of iPads in schools, offer primary school teachers access to new pedagogical resources and alternative methods which show promise in supporting early mathematics learning (see, for instance, Calder and Murphy, 2018, or Sinclair and Coles, 2017). Consequently, there is a growing body of mathematics education research that seeks to build an understanding of how the affordances of touchscreen devices, and the digital applications used with them, can facilitate mathematics learning (e.g. Attard, 2015; Sedaghatjou & Rodney, 2018).

Mathematics education researchers have also been interested in how teachers choose to implement these resources for the teaching of mathematics (e.g. Monaghan, 2004; Thomas & Palmer, 2014) and their effect on student learning (e.g. Calder & Murphy, 2018; Sinclair & Baccaglini-Frank, 2015). There is an increasing variety of digital technologies which have been specifically designed to provide valuable mathematics learning experiences. However, Ruthven (2014) found that it is teacher expertise that determines whether these technologies are successfully integrated into everyday teaching practice. For many teachers, it is a daunting, complex and challenging endeavour to skilfully leverage digital tools for effective student learning (Trigueros et al., 2014), which Sinclair and colleagues (2020) also found when primary teachers began using the iPad touchscreen application TT with their students. As Guin and Trouche (1999) explain, it is the role of the teacher to draw attention to and help students make appropriate connections between the digital technology and the mathematical knowledge they evoke.

Many elementary schools are investing in mobile technologies such as iPads; however, their integration and use in classrooms has often occurred without accompanying professional development support for teachers. Consequently, the ability of teachers to use these forms of technology effectively to enhance teaching and learning has proven to be challenging (Attard & Curry, 2012). Furthermore, Larkin and Milford (2018) have found that many of the apps downloaded onto these devices for classroom use are chosen, “without a strong conceptual, pedagogical, or methodological underpinning” (p. 12). In order to transition from traditional mathematics teaching to that which includes digital technology, it is necessary for teachers to commit to learning how to use the technology as a pedagogical tool effectively (Pierce & Ball, 2009).

Theoretical Framework

In this section, I will describe the theoretical constructs that were influential in the development of the research questions and that framed my data analysis. I will begin by outlining the key ideas taken from the theory of instrumentation (Vérillon & Rabardel, 1995) and adopted by mathematics education as part of the instrumental approach (Artigue, 2002; Guin et al., 2005; Guin & Trouche, 1999). I will then summarise the theoretical constructs of double instrumental genesis (Haspekian, 2007), instrumental distance (Haspekian, 2005) and didactical landmarks (Haspekian, 2017). Finally, I will bring the section to a close by sharing some sociocultural considerations and my two research questions.

Origins of the Theory

A psychological and sociocultural framework developed for use in cognitive ergonomics, there are two ideas from the *theory of instrumentation* on the human use of tools (Vérillon & Rabardel, 1995) that have been appropriated for use in mathematics education research. The first was the distinction between an artefact and an instrument, and the second described the process in which the artefact becomes an instrument. An *artefact* was defined as a physical or symbolic object, sometimes referred to as a tool, whereas an *instrument* is a complex psychological construct comprised of both the artefact and *utilisation schemes* (Rabardel, 1995/2002). The process whereby an artefact transforms into an instrument is called *instrumental genesis* and involves the interactions between an artefact and the individual using it. For this to occur, the user engages in utilisation schemes with the artefact in order “to develop the activity necessary to perform the functions he expects from the association of the artifact with his action” (p. 86), and in so doing, the artefact progressively *becomes* an instrument. As Bartolini Bussi (2009) explains, the instrument is specific to the individual using it and “the context within which it originates and its development occurs” (p. 153).

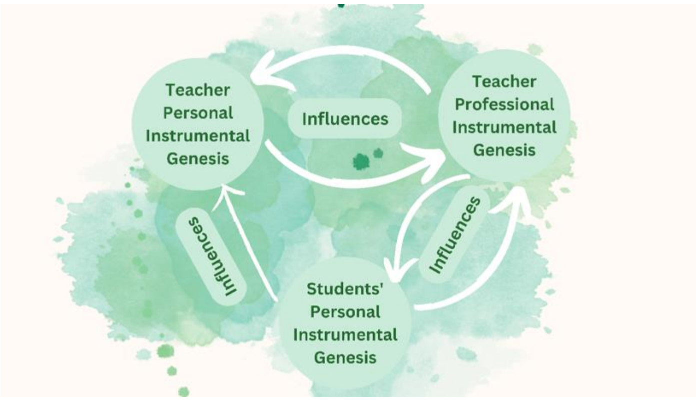
The complexity of instrumental genesis is reflected in the interactions between the artefact and the person using it; processes described by Rabardel (1995/2002) as instrumentation and instrumentalisation. During *instrumentation*, the artefact is not considered to be passive, rather, it influences and shapes the actions or thinking of the individual using it. Whereas *instrumentalisation* is the opposite process, in which each individual user transforms the artefact into an instrument that is unique and useful for their specific purposes (Trouche, 2004). These ideas proved to be useful when analysing technology-mediated learning in mathematics education and were first appropriated for use in the *instrumental approach* (Artigue, 2002; Guin et al., 2005; Guin & Trouche, 1999). As Maschietto and Trouche (2010) explain, "*Technology* can *shape teaching and learning mathematics*, while reciprocally *being shaped* by its use" (p. 39; emphasis in the original).

Double Instrumental Genesis

While Haspekian (2007) was studying the integration of spreadsheets by middle school mathematics teachers into their algebra classes in France, she observed that the process of instrumental genesis was more complex when an artefact needs to be transformed into an instrument for pedagogical purposes. She described how mathematics teachers undergo what she termed, *double instrumental genesis* (see Figure 1), a phenomenon that involves dual experiences of personal and professional instrumental genesis.

Figure 1

The Process of Double Instrumental Genesis



Personal instrumental genesis refers to the process where an artefact becomes, for the person using it, a working instrument for mathematical activity. This may be a similar process for teachers and students. Teachers, however, undergo an additional process of *professional instrumental genesis* while appropriating and developing the ability to utilise an artefact effectively as an instrument for teaching and/or student learning. "The teacher's professional genesis with the tool is much more complicated as it includes the pupils' instrumental genesis" (Haspekian, 2014, p. 254). This necessitates the teacher being able to predict and plan for students' experiences of instrumental genesis well enough to facilitate mathematics learning using the digital technology. Thus, creating the additional, professional layer of complexity for the teacher's instrumental genesis.

> The instrument that is created as a result of this process of professional genesis (for instance the 'spreadsheet as a tool to teach algebra') is different from the instrument built through a personal genesis (the spreadsheet as a tool of personal work of calculation, plotting, data treatment, etc.).

> From the same artefact, two instrumental geneses (that may have interferences/interactions on each other) lead to two different instruments. (p. 254)

A teacher's personal and professional instrumental geneses are not necessarily independent of each other, which Haspekian (2014) observed during one teacher's implementation of spreadsheets over two years. She noted that the two processes "interacted in a relational sense" (p. 256), which sometimes caused *interferences*. In the examples given, the interferences between the personal and professional instrumental geneses were related to the teacher's purpose for using a spreadsheet, whether for personal calculation or as a didactic instrument, and these interferences at times influenced the instrumental genesis of the students.

Additionally, when adopting digital technology into their pedagogical practice, teachers may experience what Clark-Wilson (2010) refers to as a *hiccup*, or "the perturbation experienced by teachers during lessons stimulated by their use of the technology, which illuminates discontinuities with teachers' knowledge" (p. 2). Furthermore, the integration of technology may also displace previous ways of teaching, and consequently, the teacher must then develop and implement new teaching methods for use with the digital technology (instrumentation). During the process of professional instrumental genesis, while teachers are transforming a digital tool into a new didactical instrument, they may also experience a phenomenon called instrumental distance (Haspekian, 2005), which will be explained further in the next sub-section.

Instrumental Distance and Didactical Landmarks

During her research on the use of spreadsheets for teaching algebra, Haspekian (2005) noted deviations, or gaps, which were caused by the impact of introducing something new (the use of spreadsheets) into previous ways of teaching mathematics (algebra). This gap, referred to as *instrumental distance*, provides a means of focusing attention on how the digital tool affects the mathematical concepts and conceptualisations (instrumentation). It is also useful for examining the changes that occur with the introduction of a digital tool, thus, providing a way of explaining teacher resistances related to the difficulties of integrating technology into their pedagogical practice. The instrumental distance must be sufficient enough that the benefits of using the digital technology are apparent for the teacher, yet small enough not to discourage the extra effort required by the teacher for incorporating something new into their practice.

Extending this idea further, Haspekian (2017) describes the importance of *didactical landmarks*[1], which function as reference markers for teachers during the development of their pedagogical practice when integrating digital technology. The theoretical tools used for examining didactical reference markers are drawn from the *Double Approach* (Robert & Rogalski, 2002), which describes teachers' practices as being complex, coherent and stable objects that "result from singular (personal) recompositions from knowledge, representations, experiences, and

[1] I will refer to didactical landmarks as teacher reference points or reference markers, as landmarks to me represent physical objects or features of land. I feel that the prior experiences or background knowledge that teachers are accessing in Haspekian's didactical landmarks are better referred to as reference points or reference markers.

individual history according to belonging to a profession" (p. 508; *my translation*). Haspekian (2017) found that if the instrumental distance is large between a teacher's new and former practices, or if the teacher's previous reference markers are largely disrupted without the consideration of new ones, then the distance will be problematic, and the creation of new reference markers is necessary. However, teachers may be able to move away from former practices more easily and quickly if they are experienced and therefore already possess various reference marks and an increased ability to adapt and adjust old reference marks to create new ones (Haspekian & Gélis, 2021).

Sociocultural Considerations

Throughout the world, society and culture heavily influence education, and conversely education impacts society and culture. Being socially situated, schools and the teachers and students within them are influenced by both internal and external social pressures which can be major factors in influencing personal development and learning. A sociocultural perspective focuses on the critical role of the social context in cognition and social development (Vygotsky, 1978). Lerman (1994) explicitly described a shifting focus that occurred in mathematics education from psychological views of individual understanding to an acknowledgement of the influence of social interactions on student learning. In mathematics education, a sociocultural perspective tends to view learning, thought and knowledge-creation as inherently social, cultural and situated (made rather than given) (e.g. Gutiérrez, 2013; Lerman, 2000).

Schleppegrell (2010) explains that, "Sociocultural perspectives focus on discursive practices and the social engagement of students. They draw on Vygotskyan frameworks that stress the interaction between language and cognition and highlight the social dimension of learning and the role of communication and participation" (p. 76). Though the social engagement of 'students' is referred to in this quotation, it can also be applied to the role of communication and participation in teacher–researcher group meetings. In the case of this article, the focus is on two teachers who were part of a larger research project that was examining the use of a new technology with students learning about multiplication. The importance of sociocultural perspectives in learning relates to their focus "on the social interactions, the language and its meaning for the individual and the groups within which [the individual] acts" (Lerman, 1994, p. 196) during the process of building understanding. Although their research was focused on the impact of technology as a boundary object, Sinclair and colleagues (2020) also described observing changes in the way that some teachers discussed the concept of multiplication as a result of interaction and group discussion in research project meetings with other participating teachers.

Extending Valsiner's (1997) Zone Theory to the context of mathematics education, Goos (2001) examined the influence of sociocultural perspectives on what she refers to as the teacher-as-learner. She outlined the intersecting influences of the Zone of Proximal Development (ZPD), the Zone of Free Movement (ZFM) and the Zone of Promoted Action (ZPA) on teachers in the role of learners. From the perspective of teachers-as-learners, the ZPD involves the potential for developing new beliefs, knowledge, goals and practices through teacher interactions with others and the environment they are in. ZFM refers to the professional context of the teacher and includes elements such as the students, curriculum and assessment standards, resources, school organizational structures and culture. ZPA includes those things that "*promote* certain teaching approaches" (Goos, 2014, p. 445, emphasis in original) and may include teacher preparation programs, teacher professional development and/or informal interaction with other teachers. Using this adapted Zone Theory, Goos examined case studies of teacher-as-learner and found that bringing teaching, learning and context together provided her, as a mathematics teacher education researcher, a better understanding of how teachers learn and how to intervene in teachers' learning and development.

Hoyles and colleagues (2004) examined the theory of instrumentation using a sociocultural perspective and suggest that ''instrumentation could be regarded as part of the process of developing participation within a community of practice, a process by which individual understanding and behaviour develops from and contributes to the collective activity" (p. 317). Additionally, the influence of the technological tool in the co-construction of knowledge is not limited solely to that of a cognitive tool, rather it is considered "a genuine mediator of social interaction through which shared expressions can be constructed" (p. 318). The tool itself influences the user's developing mathematical ideas which are also simultaneously influenced through interactions among the community of users.

The previous research on double instrumental genesis (e.g. Haspekian, 2005, 2014) has concentrated on middle school teachers using spreadsheets to teach algebra. This study, with its focus on primary school teachers, aims to expand and enhance our understanding of this complex dual process for those teachers using a mobile touchscreen technology that has been specifically designed for teaching mathematics to primary students. Therefore, the research questions that are the focus of this article are: (1) What sociocultural factors are influential to primary school teachers' processes of double instrumental genesis when using TT as a tool for teaching mathematics? (2) How do these sociocultural factors impact the evolution of primary school teachers' professional instrumental genesis of TT?

Methodological Considerations

In this section, I will begin by describing the iPad application *TouchTimes* (Jackiw & Sinclair, 2019), its central design features and the concerns that motivated its development. I will then share some background information about the research participants and the semi-structured interviews which formed the basis of this research, and, finally, I will articulate my data-analysis process.

TouchTimes

A multi-touch iPad application, TT is designed to provide users with multiplicative experiences of multiplication using an embodied co-ordination of units approach. Though TT has two complementary microworlds, *Grasplify* and *Zaplify*, each of which emphasise in visually distinct ways the different properties of multiplication, the focus of this article will solely be on Grasplify. In Grasplify, the user's hands separately take on the roles of multiplier and multiplicand, emphasising the function of each in determining the product. As children create, adjust and transform digital representations of multiplicative situations on the iPad screen, they receive immediate visual, haptic and symbolic feedback from TT.

Grasplify: Function and Design

Outside of the mid-screen vertical line that divides the display in half (see Figure 2), the Grasplify opening screen appears blank until finger contact is made on the surface of the iPad. When the user touches the screen, coloured discs (referred to as 'pips') appear at each point of contact, as if summoned by the user's specific fingertips (see Figure 3). Pips require continuous screen contact to remain visible, and if a pip-making finger is lifted from the screen, the pip (and all pips associated with it) ceases to exist and vanishes from view. Grasplify resets when all pip-fingers are removed from the screen. The numeral that corresponds to how many pips made by the user instantly appears on the upper part of the screen (see Figure 3) and automatically adjusts in response to the creation or removal of pips.

Figure 2

Grasplify's Opening Display

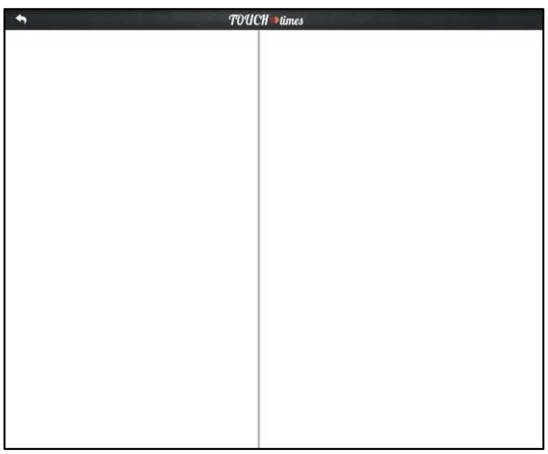
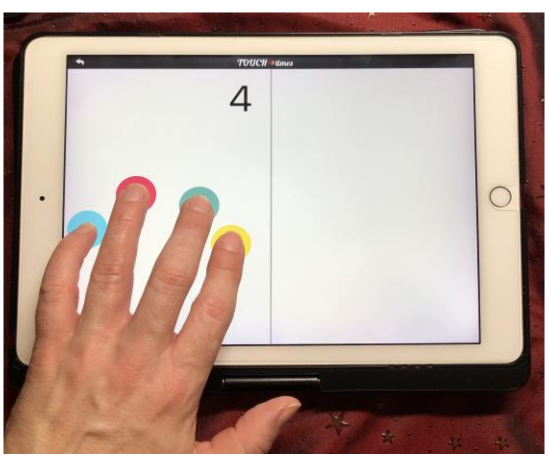


Figure 3

Creating four pips on the left side of the screen



Once pips have been established, it is then possible to create ‘pods’, or enclosed groups of pips, with the touch of a finger on the opposite side of the vertical line that splits the screen in half (see Figure 4). Unlike pips, pods do not require continuous screen contact by the user’s fingertips to remain on display. Each pod reflects the colour, shape formation and number of pips that the user is actively maintaining on the screen. After a momentary delay, the entire group of pods is encircled by a ‘lasso’ to form the product visibly, which then appears alongside the equals sign to complete the numerical expression (see Figure 4). The composition of the pods, essentially the pips within them, and the numerical expression at the top of the screen, alter instantly to reflect the creation or removal of pips on the pip-creation side of the screen, and its effect on the product.

Figure 4

Creating Pods on the Right Side of the Screen

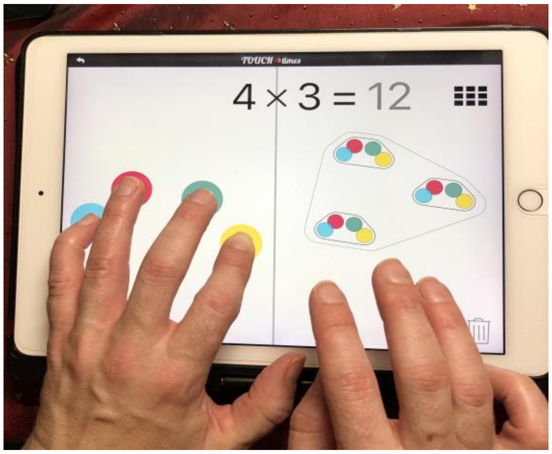
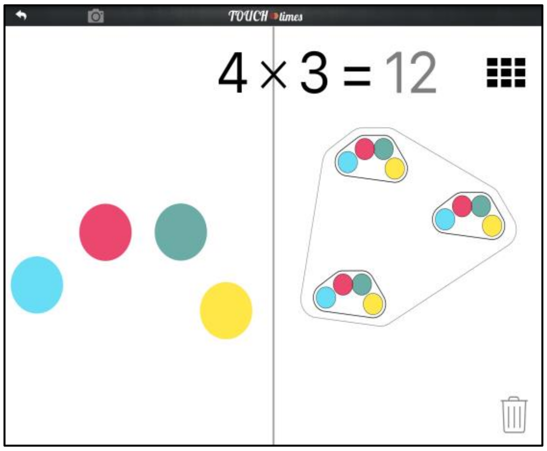


Figure 5

Two Composite Units



Designed to be symmetric, pip- and pod-creation can occur on either side of the screen, as can be seen by the progression of visuals shown in Figures 3 through 7. The pips are always produced first (see Figures 3 and 6) and the pods are then created on whichever side of the vertical line that is opposite to where the pips were established (see Figures 4 and 7). With the creation of the first pod, the numerical expression at the top of the screen adjusts to display the number of pods, and the '×' symbol becomes visible between the two numerals representing the pips and the pods (see Figure 8). The appearance of each part of the numerical expression always corresponds in time, order and direction to the initial establishment of the pips (see Figures 3 and 6), followed by the pods (see Figures 4 and 7) and, finally, the product (see Figures 4 and 8). The symbolic mathematics also visually coincides with the left/right arrangement of pips/pods and will be displayed as either pips × pods = product (Figure 5) or product = pods × pips (Figure 9). It is worth being aware that the symbols appearing at the top in Figures 6 through 9 go right to left in time, against the expectation of the writing direction in English.

Figure 6

Establishing Pips on the Right Side of the Screen

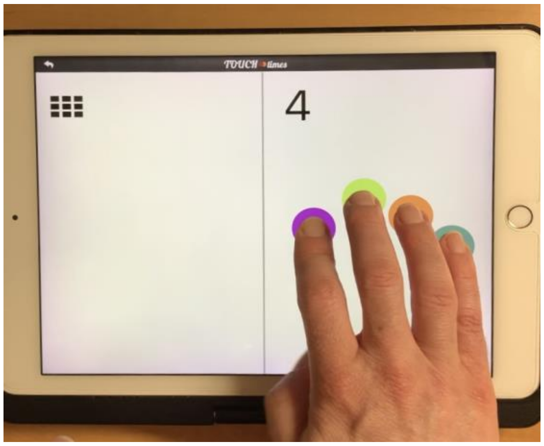


Figure 7

Creating Pods on the Left Side of the Screen

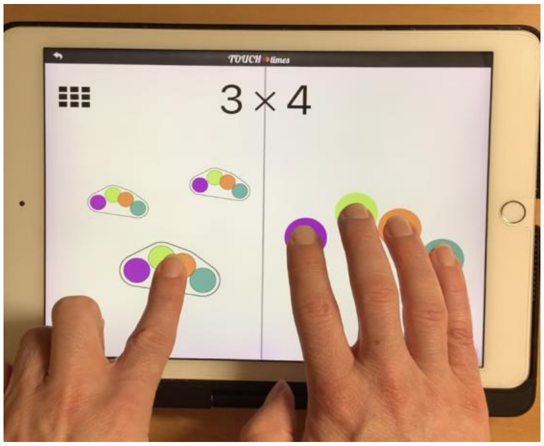


Figure 8

The Full Numerical Expression

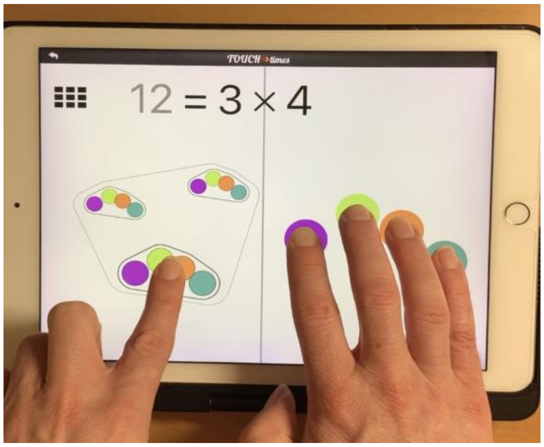
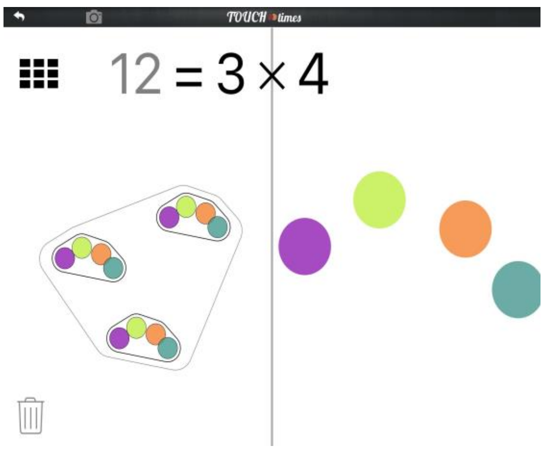


Figure 9

Two Composite Units



Grounded in approaches to mathematics that are based on measurement and ratio, Davydov's (1992) double change-in-units approach describes multiplication as involving "a count of a [larger] unit for which a relationship to another, smaller unit, is already established" (p. 12). When creating composite units, this requires the unit of measure (the multiplicand) be established before the unit quantity (the multiplier). These ideas were influential in the design of Grasplify, where the pods (the multiplier) are a quantity dependent on the establishment of pips (the multiplicand) and serve as the unit of measurement for the final product. The co-ordination of Davydov's double change-in-unit process is embodied by this microworld, where the first unitising is seen when the pips are established (in Figures 3 and 6, there are four pips), the second unitising takes place as the pods (a unit of units) are created (in Figures 4 and 7, there are three pods), and the product is reflected by the encircling of the pods by the lasso (in Figures 5 and 9, there are two composite units), which is when the third, and final, unitisation occurs.

This multiplicand × multiplier ordering is intentionally incorporated into Grasplify, though it is the opposite of what elementary teachers in British Columbia (BC) usually find in textbooks and teaching resources (and, potentially, their own student school experience). This ordering of factors, where the multiplicand precedes the multiplier in time, is reflective of the Davydovian approach to multiplication, where the *unit quantity* must be established prior to determining the number of units. The chronological ordering of the appearance of each factor is important and is therefore asymmetric.

The Research Participants

The data for this qualitative study was gathered during semi-structured, 60–80-minute teacher interviews that were conducted via Zoom between June and August 2021. This article focuses on Leah and Rachel (pseudonyms), two of the four teachers who were interviewed as part of my doctoral research (Bakos, 2022). All of whom were voluntary participants in another larger, multi-year TT research project, in which the author was also part of, and were implementing TT as a teaching tool for multiplication with grade 3 or grade 3–4 students (8–9-year-olds), which is when multiplication is initially introduced to students in the BC mathematics curriculum. Rachel and Leah are both primary school generalist French-immersion teachers, with nine and twenty-four years of classroom teaching experience respectively, at the time of being interviewed. Though not a requirement to teach in BC, Rachel has a master's degree in numeracy and Leah has a master's degree in literacy.

Both teachers were part of a larger, multi-phase project (see Sinclair et al., 2020) that was comprised of 11 teacher volunteers working in eight different schools in Metro Vancouver, two professors and two doctoral students (one of whom was the author). During the course of the 2018–2019 school year, there were five teacher–researcher group meetings. During the first meeting, the project aims were explained, and teachers were introduced to TT, which was still in the development phase and not yet available to the public. Teachers were given time for free exploration of Grasplify, and then the researchers engaged them in some pre-developed tasks as a way of introducing various functions of the application. At this time, Zaplify was not yet fully developed and so teachers only had access to Grasplify. Throughout the meetings that followed, teachers and researchers discussed the various features of TT, the mathematical intentions behind the design of Grasplify, and later, Zaplify, as well as the purpose behind each of the tasks shared. New tasks were introduced for teachers to explore and potentially use with students. As teachers began to use TT and its associated tasks in their classrooms, a significant portion of the meetings became devoted to sharing these experiences with the group and providing feedback about the technology, asking questions about the mathematics it presented and improving existing tasks and sharing ideas for the creation of new tasks.

There were five teachers that volunteered to have the research team come into their classrooms to observe their implementation of TT with their students. The author was able to observe three of these teachers while they were using TT as a pedagogical tool for mathematics on multiple occasions and asked them if they would consent to being interviewed about their experiences. All three teachers agreed, along with a fourth teacher who had expressed interest in being interviewed but due to the COVID-19 pandemic, the interviews were delayed and did not occur until over a year later, in June and August of 2021. Each of the teachers was initially interviewed individually and was later paired up for an additional interview. The pairings were made based on when the teachers were available to meet, and Rachel and Leah were not interviewed together in the partner interviews.

Rachel and Leah were chosen as the focus of this article because of the explicit nature of the sociocultural influences they each described in their interviews. During the larger multi-phase TT research project, there were whole group teacher–researcher meetings held, where the 11 teachers and four researchers involved would often clarify or expand upon the ideas shared within the group. Hoping to encourage this type of elaboration or clarification of experiences and ideas in the interview setting, each teacher was paired up for a second interview. During the interviews each teacher was asked to share their recollections of their initial exposure to TT and its way of presenting multiplication, as well as any memories of learning to use it themselves. We also discussed their experiences of implementing TT as a pedagogical tool for teaching multiplication, their observations related to the mathematics and how students interacted with multiplicative ideas when using this digital technology and the mathematics it presents. The teachers also shared any thoughts or concerns about TT as a technological tool, either of its models of multiplication, or any other observations about how students used the technology or what they learned about multiplication. In the next section, I will describe the process of analysis used when examining this data.

Data Analysis

All six interviews were recorded and transcribed in their entirety, though I draw from only the interview transcripts that involved Leah and Rachel. Initially, I was curious about the effects of TT and how it materialises multiplication on the teachers and how each of the teachers accommodated TT into how they thought about and taught multiplication. While studying the interview transcripts though, I also noted that all four teachers mentioned, some of them describing in detail, the significance of other people on their adoption and use of TT into their teaching practice. In addition to the influence of students on teachers' processes of double instrumental genesis, which has been previously noted by Haspekian (2017), I found that the influence of other teachers was also impactful on Leah's and Rachel's processes of double instrumental genesis (Bakos, 2023).

Given the influence of others that had emerged in the interview data from multiple teachers, I wanted to better understand how these sociocultural aspects impacted the teachers' double instrumental genesis. I then further analysed the interview transcripts with the intent to: (1) identify the sociocultural factors described by the teachers as being influential to their double instrumental genesis of TT; and (2) to highlight specific instances of interactions with others that were influential to the evolution of the teachers' professional instrumental genesis. After categorising the instances of personal interactions that were described by Leah and Rachel as being influential to their implementation of TT in their mathematics teaching, there were two categories that emerged. These were the influence of the teacher–research group from the larger TT project and the influence of their students. When engaging with qualitative data, MacLure (2010) describes a potentiality or 'glow' that can be felt emanating from the data that "seems to reach out from the inert corpus (corpse) of the data, to grasp us" (2013, p. 228). Recognising that the richness of an experience or insight shared by an individual teacher has much to convey, the instances that have been shared in the results that follow were chosen with this in mind, as well as for their ‘glow’.

Results

During the individual and paired interviews, while trying to get a sense of how each teacher’s thinking about multiplication and/or about TT had evolved as a result of their personal process of instrumental genesis, the influence of others was specifically mentioned and described in different ways by the teachers as a critical factor. From the time of their initial exposure to TT to the point of being interviewed, each teacher had implemented and used this digital technology as a pedagogical instrument for teaching multiplication. Leah and Rachel both described the impact of other people as being significant during their adoption and integration of TT into their professional practice. In this section, I will examine the personal interactions that were described by each teacher as being valuable to understand better how the influence of others affected each teacher’s process of double instrumental genesis.

The Influence of the *TouchTimes* Teacher–Researcher Group

Rachel and Leah both mentioned how valuable it was to be involved in the larger TT teacher–researcher project, and to have a chance to interact with other teachers who were also implementing this digital technology as part of their mathematics teaching. The influence of the TT teacher–researcher project was described by both teachers as being fundamental in helping them overcome their initial difficulties with understanding and implementing Grasplify as a pedagogical tool for teaching multiplication. I will first examine the experiences shared by Rachel, before turning to Leah, who discussed overcoming her difficulty with the multiplicand × multiplier ordering of Grasplify.

Rachel: Initial challenges and becoming reflective about teaching multiplication

When asked about her initial impressions of TT, Rachel described how she had missed the first TT teacher–researcher meeting and, as a result, she did not quite understand some of the tasks given during the second meeting. She compared her prior experiences with *TouchCounts* (Sinclair & Jackiw, 2011) with her initial experience with Grasplify. “*TouchCounts* I get. […] And I know it’s also for a lower grade level, but still, multiplication at the grade two to five level shouldn’t be that complicated. It wasn’t intuitive.” Initially finding TT less intuitive to use and harder to think of tasks by herself than she had previously found when using *TouchCounts* as a teaching tool, Rachel described this as challenging (Bakos, 2023). “I found it harder to use and harder to think of things on my own. […] I was able to think of other things I could do with *TouchCounts*, whereas with *TouchTimes*, I still really needed the teacher guide to assist me.” She found herself relying heavily on the TT task ideas that were developed by the project researchers and shared during the teacher–researcher meetings.

Rachel mentioned how beneficial the research group meetings were in helping her overcome her initial challenges in figuring out how best to implement Grasplify in her classroom. The opportunities for small- and large-group discussions in the TT research project meetings allowed for teacher participants to share their experiences using the digital application in their classrooms, to describe the tasks they are tried out and how these were implemented in their grade level and what successes or challenges their students were having while learning multiplication using TT. The instrumental distance for Rachel was large at this point; her personal instrumental genesis was influenced by not quite understanding some the tasks shared at the second meeting, having missed the first one, and she did not feel comfortable implementing or utilising Grasplify without further support. Rachel was very explicit in sharing the importance of being part of the larger research project. “Just understanding how it [Grasplify] worked and seeing what other people were doing […,] it was just nice to get a sense of what people were doing in their classrooms and what I might be able to try.”

Rachel’s prior experiences teaching mathematics with *TouchCounts,* a digital application designed by the same developers, contrasted sharply to her initial experiences with TT. The instrumental distance was large and in order to use TT as a pedagogical tool with her students, Rachel relied on the pre-developed tasks for use with Grasplify and on the experiences and ideas shared by the other teacher participants as a way of narrowing this gap. The shared experiences of other teachers enabled Rachel to create *‘shadows’* of her own didactical reference markers to rely on initially until she could solidify these through her own teaching experiences with of Grasplify.

When first introducing TT to teacher participants of the larger project, the research team shared that one of the project goals was to improve multiplicative thinking in students. Often students are introduced to multiplication through repeated addition models, which becomes problematic later when students are required to think multiplicatively in middle-school mathematics (e.g. Siemon et al, 2005). Grasplify’s design and the tasks that were developed for use with it, expose students to ways of thinking about multiplication that are more multiplicative in nature. As a result, Grasplify’s way of presenting multiplication and the importance of highlighting multiplicative ideas continued to be a topic of discussion within the TT teacher–researcher group meetings which proved to be impactful for both Rachel and Leah.

When interviewed, Rachel explicitly described how she developed a deeper understanding of the multiplicative way that Grasplify presents multiplication and her recognition of the importance of students developing the ability to reason multiplicatively. She explained, “I think it was like coming to the sessions and listening and having the discussions [in response to questions prompted by the research team]” that prompted her to begin rethinking the way she approached teaching multiplication. “I think it definitely did get me thinking about the properties of multiplication more than I had if I weren’t using it.” The task ideas shared by the researchers and the discussion amongst the group about TT and the way it materialised multiplication influenced how Rachel implemented it with her students and drew her attention to the mathematical affordances of the tool.

> For me, using *TouchTimes* or listening to Nathalie, or being part of those discussions, it was like, “Oh, we really need to be showing students that multiplication is more than just this”. So, in terms of impacting my teaching, whether using the app or not, I guess it made me think about, like the words I’m using or what I’m saying, or how I’m showing multiplication or teaching it or what, you know, whatever the activity is that the students are doing, that I need to be aware of how I’m representing multiplication.

Rachel then became more intentional in engaging her students with tasks that highlighted properties of multiplication and pushed student explanations and thinking beyond additive thinking towards multiplicative reasoning. She shared how, “If I were just using it [TT], not being part of a cohort of teachers who are meeting and talking about it and really thinking about it, would it affect me as much? Probably not, but it did.” Although the multiplicative model presented by Grasplify is what provoked these conversations, it was the influence of the teacher–researcher group discussions that prompted Rachel to rethink how she could teach multiplication with Grasplify.

Leah: Context, commutativity and relationship

For Leah, the multiplicand × multiplier = product ordering of Grasplify caused a significant disturbance, though there were two very different sociocultural influences that proved beneficial to her process of double instrumental genesis. The first, which will be addressed in this section, was the TT teacher–researcher group and the second, which will be examined in the section that follows, was the influence of Leah’s students themselves.

The opportunity to share questions and discuss possible answers amongst the group was a significant factor in Leah’s process of double instrumental genesis. She explicitly stated, “I think that teachers being able to work with this together is also really beneficial. […] The ideas that come out of it are pretty big, especially when somebody is new to it [TT] too.” Leah remembered thinking to herself at one of the teacher–researcher meetings, “Okay, there's something missing [in reference to her difficulty with Grasplify’s representation], these are people who really know a lot of math and number. I’d better think about my thinking. It bothered me that there was another way that I hadn't thought about.” The presence of ‘knowledgeable others’ made Leah pause and question her own understanding of multiplication.

At the second TT teacher–researcher project meeting, Leah wanted further reassurance, stating that she was still confused about “the opposite way that the app is looking at it [multiplication] than some of us are used to teaching it.” Another teacher in the group, Monica (a pseudonym), explained the importance of context in relation to the numerical expression and its multiplicative model.

> For me, it’s context, right? So, that’s when you want to make sure that the groups-of works. Like if you’ve got, there’s a difference between if you’re looking at cars with four wheels and you know they [students] know how many wheels. There’s a difference between two times four and four times two in that case.

Monica went on to question whether this idea that 3 × 4 had to be three groups-of four was a “real rule”. Then provided an example of a multiplicative situation where the groups-of scenario would be influenced by which factors were easier to visualise, rather than their position in the numerical expression.

> The other thing I was going to say too is if you have a hundred times three, are you going to expect like you’re doing a hundred groups of three? Or are you going to visualise three groups of a hundred?

This explanation and the examples provided by Monica in November of 2018 made an impact on Leah, who shared the essence of the cars and wheels example with me during an interview nearly three years later. She explained how helpful Monica's example was and that, "it [the order] does matter, sometimes. So, if you're talking about something that has a set amount of things, like cars have four wheels. Groups-of matters in that language." Leah already had didactical reference points for thinking about groups-of and this example, and its acknowledgement that context sometimes did matter when creating a multiplicative model, helped her accept that, in most multiplicative situations, "It doesn't really matter because it all equals the same thing".

The instrumental gap was becoming narrower because of this discussion with another teacher, who provided an option to think of multiplicative models in relation to context. Though this notion of visual representations being dependent upon context was not explicitly connected to the ordering of the numerical expression (which is what created the disturbance for Leah, initially), it was still effective in transforming her thinking. Perhaps what was helpful for Leah was the acknowledgement that, unless a certain context demands that a multiplicative model be represented in a specific way (as in the cars and wheels example), mathematically the ordering of the factors is not important.

In reconciling for herself that the multiplicand × multiplier = product ordering of Grasplify should not matter, Leah also referred to another didactical reference point, which was her knowledge of commutativity. She explained how this property of multiplication was helpful as a way of making sense of the irrelevance of the ordering of the factors. Though her understanding of commutativity enabled Leah to bridge the instrumental distance between Grasplify's way of modeling multiplication and her previously held groups-of model, commutativity is not applicable in the context of Grasplify where the multiplicand (represented by the pips) and the multiplier (by the pods) have differing roles and are not interchangeable. Though the product is the same for $a \times b$ and $b \times a$, the emphasis in Grasplify is not on the product itself: rather, it is on the procedural establishment of the pip(s), then the pods and the subsequent 'spreading' of the pips across each of the pod-units. Despite commutativity not being relevant to Grasplify, having the knowledge of this property was helpful in permitting Leah to disregard her previous belief that the only correct way to represent 3 × 4 was as three groups-of four.

When interviewed three years after starting with TT, Leah explained that "I was so stuck on this groups-of thing and then I started thinking about, well, what does multiplication mean? So, it really changed my thinking about what it [multiplication] means." She went on to describe further the transition that she had experienced, which I believe nicely reflects the intertwined nature of her personal and professional instrumental geneses. "If I really believe in […] how this [application] works, and what multiplicative thinking means, it doesn't matter what happens next. It's what happens in their [her third-grade students'] thinking."

Leah also spoke about how, as a result of the car and wheels example and the subsequent discussions within the teacher–researcher group about how to describe the multiplicative model in Grasplify, she now used different ways of speaking about multiplication with her students.

> So, to me, you can do the three cars with four wheels, three groups of four. I still do use that language sometimes, but then you can teach, "Well, on *TouchTimes*, it's actually thinking this way: three, five times." And so, when you're teaching that, you are thinking about it in that way. And this way, it actually deepens the understanding, even more. So, the conflict I had was actually a gift.

The instrumental distance between Grasplify's way of representing multiplication and the groups-of approach previously used by Leah when introducing multiplication in her class provoked her to question her own understanding of what multiplication means. Her previous ways of teaching multiplication had come to influence her own thinking about multiplication. When confronted with Grasplify's model, which was not consistent with her understanding of multiplication, Leah's personal instrumental genesis was challenged. The influence of other members of the teacher–researcher group was instrumental in Leah's ability to overcome this challenge. As a result of her personal instrumental genesis of Grasplify becoming stronger, her professional instrumental genesis strengthened as well. She began to pose the questions that she had asked herself in reconciling her understanding of multiplication to her students.

> I'd say, "Well, what does this multiplication [draws an '×' sign with her hand in the air] mean? Well, can it mean this, this many times? Or this many groups of? Can it mean both? And why can it mean both? And I got them [her students] to start thinking about that and showing that. So, for me, it means both. It can mean three times five, it can mean three groups-of five or it can mean three, five times. It means both. Depending on the context, depending on the wording.

Leah's reference to context and the language used to describe a multiplication, both ideas which emerged as topics discussed during project meetings, illustrate how Leah's process of double instrumental genesis was positively impacted by her interactions with other members of the TT teacher–researcher group. Like Rachel, Leah also mentioned the impact of having the research team observe in her classroom, though what she described as being beneficial were the short conversations with the team following her teaching. These occurred immediately after the lesson when the students were outside for recess and the research team was packing up to leave. Leah would often share what she had noticed or thought went well in the lesson and the research team would share their observations in response to Leah's comments. When interviewed, Leah described how valuable she found these conversations with the research team:

> This is why this needs to be done like this [in reference to the group coming in to observe her teaching]. Like, I think you guys coming in, because you're, you think about these things because you're thinking about the app all the time. And then having these conversations after, in the context of what happened, like what I just got out of this is like whoof, mind blowing. [She laughs.] Like it's making me think about the whole thing in a whole different way […] I'm thinking about the colours, I'm thinking about the pips and the pods, and the importance, like, you know how I got over the groups-of? And I got like, really over that. Now I'm like, it's *really* about the relationship.

Moving beyond her initial challenge with the multiplicand × multiplier ordering of Grasplify occurred in multiple steps for Leah. Her first step occurred when another teacher explained the importance of context and the next step happened when Leah used her knowledge of commutativity to reconcile that $a \times b$ can be a groups-of b or b groups-of a. There is another significant factor, which will be addressed in the next section, however, the comment above is an important step forward in Leah's understanding of Grasplify's way of materialising multiplication. A step that occurred during a spontaneous conversation between the researchers and Leah after observing her teach a lesson using Grasplify with her students. The influence of others throughout this process had a significant impact on Leah's understanding of multiplication and her ability to use TT effectively as a pedagogical tool for teaching mathematics to her students.

The influence of the students

A second sociocultural aspect, explicitly mentioned by both teachers as being influential to their evaluation of TT as a pedagogical tool, was the students themselves. Leah and Rachel both described different ways that student comments, actions and learning were significant to their evaluation of TT. This aspect of evaluating the effects of their teaching and the digital tool chosen to address their mathematics teaching goals is a crucial consideration when examining the evolution of the double instrumental genesis process the teachers were engaged in. Additionally, each teacher's assessment of the effectiveness of the digital technology was an important part of the teacher's double instrumental genesis, informing their choice to continue using this digital tool for teaching multiplication or to abandon it in favour of other teaching strategies or tools.

In the section that follows, I return to Leah's story of how her students were instrumental to her acceptance of the multiplicand × multiplier ordering of Grasplify which had caused her ongoing discomfort. After that, I will examine how Rachel's processes of double instrumental genesis were impacted by the students she was teaching.

Leah: "The kids are okay with it"

As Leah began using Grasplify in her classroom, her students' successful use of the digital tool and understanding of the mathematics it presents was deeply compelling for her. She was very forthright in sharing that the children's reactions to Grasplify were quite influential: "one thing that affected me is the conversations I had with the kids as a teacher." Unhindered by the rigid view of multiplication models that must reflect a multiplier × multiplicand ordering that was so troublesome for Leah herself, her students easily used the language of 'three, five times' or 'five groups-of three' for 3 × 5 when using Grasplify. She observed that her students "didn't know any different, and so they were understanding it [the ordering] the way it was, and it didn't matter."

Given that Leah's didactical reference markers were based upon her previous ways of referring to multiplication through an $a \times b$ numerical expression which was explained as *a* groups-of *b*, she was surprised when her students used the term "groups-of", even though she had been very intentional about not formally teaching her students this expression. "Interestingly enough […] they still talk about groups-of, even though the sentence is the other way around. So, they'll say 'oh, that's three groups-of two', even though it [the numerical expression] says two, three times."

After observing her students speaking about multiplication in terms of groups-of based on the visual model depicted rather than the order of the numerical expression, Leah questioned her own thinking about multiplication. She was curious about why the ordering in Grasplify was so problematic for herself but was not for her students. Therefore, she began to explore the words used to describe the multiplicative scenarios created using Grasplify more intentionally with her students. Leah shared an example of how she approached this:

> So, I think it was like one times seven I put on the board and then I said, “Okay, make that.” [using Grasplify] Yeah and then I said, “Is seven times one the same thing on *TouchTimes*?” [see Figure 10] And of course, because of commutativité, how I always say it in French, commutative, commutative property. They all said, “Well, it’s the same thing”. And I said, “But does it look like the same thing?” So, then I put one times seven on the board. “Make that on your *TouchTimes*. [see Figure 11] What does that mean on *TouchTimes*? Talk to me in *TouchTimes* language.” It was funny how a lot of kids went, “Well, it's one group of seven”. Finally, one kid pipes up, “No, it’s not. It’s one, seven times." So, it was so fascinating because it made them *look* at that. […] Some of them got confused by that, but we changed it with two. If you change it with that, you’ll see. There you go. There’s two, seven times. [see Figures 12 and 13]

Figure 10

7 × 1 or seven, one time

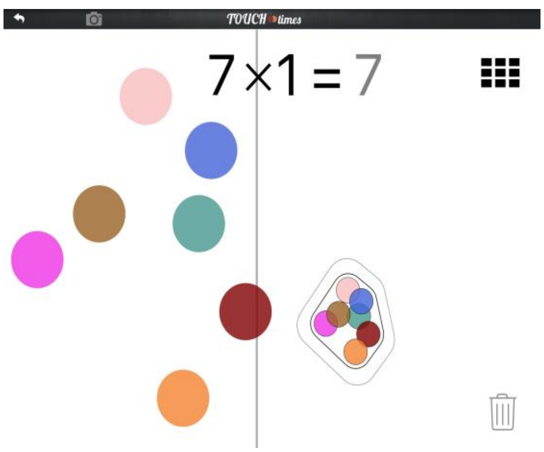


Figure 11

1 × 7 or one, seven times

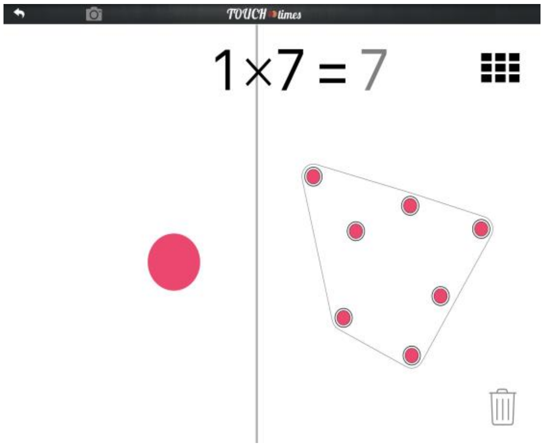


Figure 12

7 × 2 or seven, two times

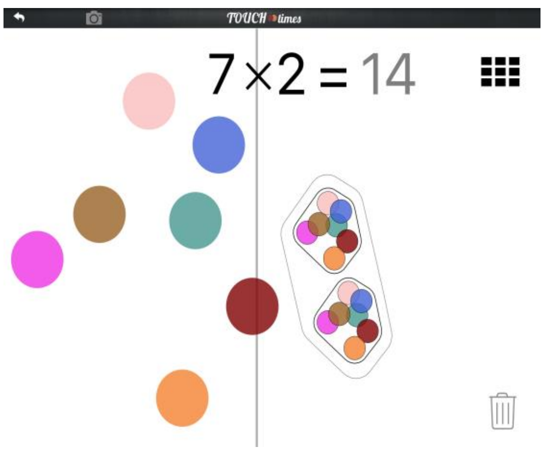
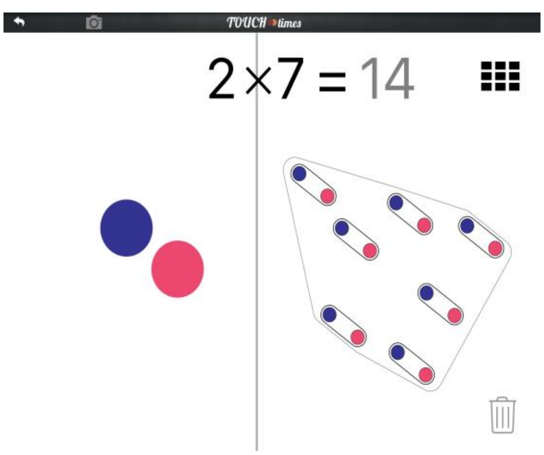


Figure 13

2 × 7 or two, seven times



In one of the video-recordings taken during a lesson observation in Leah's classroom, she explained to a member of the research team that her students were "okay with the order". Curious to know more, I shared with Leah the video-clip containing this moment in a later interview and asked her to elaborate further on this comment. She explained:

> That's not the old-fashioned groups-of. […] This whole groups-of thing, the way the order of the sentence is. Which still, people I've showed it to can't get past it. But the kids are okay with it. It's working for them, this five times. So why are we so stuck on it? Right? That's what I meant by it. See, the kids are okay with this, they're learning, they're understanding it. I was amazed that that child did it exactly how *TouchTimes* would do it without even a flinch. […] She does two, three times. Not two groups of three. Whereas she's able to look at this and go, "Oh look, I have two, three times". Exactly like it would be up there on *TouchTimes* and make that connection quickly without prompting.

Although Leah's personal instrumental genesis of Grasplify was significantly impacted by her discomfort with the way the application modelled the numerical expression created by the user, this did not prove to be an obstacle for her students, nor for the development of her professional instrumental genesis.

Rachel: Persuaded by student success

Rachel's process of double instrumental genesis was also influenced by her students' use of Grasplify. During our individual interview, Rachel noted that her students were engaged with the digital technology tasks for much longer than she had expected. She observed that the student pairs engaged in the tasks without experiencing the same difficulty she had initially.

> A lot of them got it kind of faster than I thought they would too because I, you know, I'd been to the meetings and was thinking like this app isn't like quite as intuitive as *TouchCounts*. Let's see how this goes and their understanding, I think, was better than I thought it was going to be, kind of right away. And a lot of them had a lot of success with the tasks we gave them. […] They got there quicker than I thought they would.

Student success with the Grasplify tasks motivated Rachel's continued use of the application, and she became more purposeful in drawing attention to multiplicative ideas, such as covarying and spreading (see Bakos et al., 2022 for examples of this). Through her ongoing use of TT, Rachel learned more about Grasplify from her students while they explored the tasks given and consequently, her professional instrumental genesis continued to grow. Similar to Leah, Rachel's personal instrumental genesis developed in a deeper way while using TT with her students.

Discussion

For Rachel, there was a large instrumental gap between *TouchCounts*, another mathematics iPad application with which she had previous teaching experience, and *TouchTimes*. When first introduced to TT, she did not find it to be intuitive to use and had difficulty thinking of ways to use it with her students. After missing the first TT teacher–researcher group meeting, Rachel described not understanding some of the tasks given at the second meeting, which negatively affected her personal instrumental genesis of the tool. Lacking didactical reference markers for TT, she relied on the ideas shared by other teachers in the teacher–researcher group meetings and the pre-developed tasks shared with the group, in order to feel comfortable enough to implement Grasplify in her classroom.

Both Rachel's personal and professional instrumental genesis were significantly impacted by her engagement with the TT teacher–researcher group. The discussions and group interactions also influenced Rachel's thinking about multiplication and, consequently, her way of teaching it using Grasplify was also shifting. She was more intentional in her teaching about different ways of representing multiplication and was working to provide her students with opportunities to think multiplicatively through the tasks she implemented with Grasplify.

Rachel's personal instrumental genesis of Grasplify and her understanding of its representations of multiplication were becoming stronger, which enabled her to leverage this understanding into her teaching. What she believed students should understand about multiplication was changing as she learned how the affordances of TT could be used to assist students in understanding multiplication beyond models of repeated addition. The sociocultural influence of the TT research group was essential in Rachel's initial construction of 'shadow' reference markers used to bridge the instrumental distance between her previous methods of teaching multiplication and those using Grasplify. Once she had bridged this distance, Rachel could use her own teaching experiences to build her didactical references, thus narrowing the instrumental gap.

From the outset, Leah's instrumental distance was large between Grasplify's multiplicative representation and the model that she had internalised and been using for many years to teach multiplication. Her didactical reference markers for explaining the meaning of numerical multiplication expressions were based on $a \times b$ being *a* groups-of *b*, as it was described in the textbook resource used by her school district. Therefore, the arrangement of pips and pods, and their corresponding numerical expression as it was created in Grasplify, was "backwards" to the groups-of model that Leah used for thinking about, and teaching, multiplication. The digital technology and its way of materialising multiplication conflicted with Leah's previous reference markers for multiplication. This caused a significant disruption to her process of double instrumental genesis and ultimately provoked Leah to reconsider what multiplication means.

However, despite her own personal instrumental genesis being challenged, she proceeded to implement Grasplify in her classroom and was able to instrumentalise the digital technology to serve her didactical intentions. As a result, her professional instrumental genesis developed much more quickly – and in a stronger way – than her personal instrumental genesis initially did. The second impact on Leah occurred when her class did not have the same difficulty with the ordering that she had experienced herself. When her students seemed to understand and were able to talk easily about the multiplicative models created in Grasplify, it caused Leah to question, "So why are we [elementary teachers] so stuck on it?"

Possibly because TT is a digital tool designed specifically for teaching mathematics, Leah's experience of double instrumental genesis seemed to begin with little attention paid to her personal genesis of TT. Instead, she immediately focused on how Grasplify was backwards to her previous ways of thinking about *and* teaching multiplication. As a member of the teacher–researcher group, however, Leah committed to implementing TT in her classroom, which created an opportunity for the development of her professional instrumental genesis.

Ultimately, the explanation shared by another teacher in the TT teacher–researcher group and her students' lack of difficulty with Grasplify's ordering were the two factors that proved to be most influential in Leah's ability to accept the multiplicand × multiplier ordering in Grasplify. This narrowing of the instrumental gap for Leah occurred because of interacting with others in the teacher–researcher group, as part of the process of implementing the technology as a pedagogical tool for teaching mathematics to her students and because of her observation that her students were not experiencing the same difficulty with the ordering that she had. Leah's success with instrumentalising Grasplify to serve her didactical objectives is reflective of the development of her professional instrumental genesis of the tool. Her successful use of Grasplify with students prompted Leah to question her own thinking and understanding of multiplication.

When interviewed, Leah described how the influence of other teachers in the TT teacher–researcher group, the impact of having researchers observe her teach using Grasplify and the conversations we had about the lesson afterwards and the effect of her students' success with the technology and the mathematics it presented all caused her to really think about the colours and the pips and pods in Grasplify. Multiplication, for her, was no longer solely about groups-of; instead, "it's really about the relationship". Leah's process of double instrumental genesis was not straightforward; rather, it involved a complex interaction between and amongst multiple factors: TT itself, Leah's prior ways of thinking about and teaching multiplication, her students, the teacher–researcher group and the mathematics itself.

Conclusions and Limitations

Throughout this article, I have attempted to draw attention to the sociocultural factors that were described by Leah and Rachel as being influential during their appropriation of TT as a pedagogical tool for teaching mathematics. While examining these instances, I have explained how they were beneficial in further developing Leah's and Rachel's processes of double instrumental genesis. The teachers described how other members of the TT teacher–researcher group influenced their ability to utilise this digital application effectively as a tool for teaching mathematics and how witnessing the success of their own students increased their willingness to accept TT as a viable pedagogical resource.

Leah and Rachel both highlighted the importance of being able to engage with others who were familiar with or using TT as part of the project (both teachers and researchers). Not only was Rachel able to overcome her initial difficulties with how to use Grasplify as a teaching tool, but she also talked of an increasing recognition of the importance of developing multiplicative thinking in her students. These results were facilitated by her interactions with the other teachers who were sharing their experiences using TT and by her involvement with the research team.

Leah’s ability to move past the multiplicand × multiplier ordering in Grasplify was influenced by the explanation of another teacher in the TT teacher–researcher group and as a result of her students not having the same difficulties with this that she had had. The influence of others can not be underestimated in the development of Leah’s double instrumental genesis and the narrowing of the instrumental gap between her former ways of thinking about and teaching multiplication, and the relational and embodied experiences of TT.

The processes of personal and professional instrumental geneses that were engaged in by Leah and Rachel were not linear: but were very much intertwined. Their understanding of, personal comfort with and confidence in using Grasplify improved as a result of their interactions with other teachers involved in the project and after observing their students’ successful use of the digital technology. The instrumental distance continued to narrow for Leah and Rachel after witnessing the mathematical understanding of multiplication that their students were demonstrating and were able to explain while using Grasplify. These sociocultural influences were significant as each teacher began to recognise, appreciate and leverage the interactive opportunities with multiplication that Grasplify could offer to benefit student learning. The influence of the students’ successful engagement with the embodied and relational models of multiplication provided by TT was significant in contributing to the progression of each teacher’s professional and personal processes of instrumental genesis.

Previous research by Sinclair and colleagues (2020) found that TT acted as a boundary object when teachers and researchers come together to share pedagogical and/or mathematical ideas. This finding was also confirmed by the experiences described by Leah and Rachel during their individual and paired interviews. Grasplify and the mathematics it presented acted as a mediator in the TT teacher–researcher project and, in so doing, provided an additional opportunity for the development of each teacher’s double instrumental genesis of the digital technology.

In returning to my first research question, namely how TT affected the teachers’ process of double instrumental genesis, although TT and its way of presenting multiplication clearly did influence and shape the actions and thinking of the teachers, I would argue that this process of instrumentation was not solely individual. In other words, it did not occur exclusively between the digital tool and the individual teacher using it. Rather, there was a complex intra-action (Barad, 2007) occurring within TT, the individual teacher and the collective TT teacher–researcher group. The reverberations of the effects of this digital tool affected individual teachers, as well as the group as a whole. This is particularly apparent in the multiplicand × multiplier situation that caused such difficulty for Leah and consequently emerged within the larger group as well.

When examining my second research question about how teachers instrumentalise or accommodate the instrumental distance between TT and their former ways of teacher multiplication, sociocultural aspects emerged as influential here too. The opportunity to share with other members of the research project successes, challenges and questions related to TT, the mathematics it presents and implementing it with students was explicitly stated as beneficial by both Leah and Rachel. The instrumental gap was narrowing as they built ‘shadow’ didactical reference markers based on the shared experiences of other teachers prior to building their own didactical reference markers through their own teaching with TT. The influence of the students themselves was also a significant part of Leah's and Rachel's evaluation of TT as a pedagogical tool that was effective and worth continued use or as something to be abandoned. When these teachers saw that their students were understanding the multiplicative models afforded by the technology, this impacted their professional instrumental genesis as they came to accept TT as a valuable tool for teaching mathematics. Additionally, in the case of Leah, when the students were not having the same mathematical difficulties that she had experienced, it caused her to rethink her definition of multiplication and how to teach it, causing growth in her personal instrumental genesis of the tool.

The sample size of this study, with its focus on the experiences of two primary school teachers in the context of British Columbia, Canada, was very small. Thus, the results described are specific to these individual teachers and the situations in which they taught mathematics using TT and, therefore, generalising or transferring the results of this study must be done with caution. Sociocultural influences emerged as being a significant part of these two teachers' processes of double instrumental genesis of TT, yet influences of other teachers have not previously emerged in the research using this theoretical notion. More research is needed to understand better the role of sociocultural influences on primary school teachers who are implementing technology into their mathematics teaching. Additionally, the teachers that formed the basis of this research had been teaching from 9–24 years, and therefore had experience in teaching multiplication and Rachel also had experience using digital technology as part of her mathematics teaching. It would be worthwhile exploring what the experiences of novice teachers, or those without prior experience implementing digital technology into their mathematics teaching, would be. This could provide valuable information about what supports teachers need in order to effectively implement technological resources into their mathematics teaching.

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References

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematics Learning*, *7*(3), 245–274. doi:10.1023/A:1022103903080

Attard, C. (2015). Introducing iPads into primary mathematics classrooms: Teachers' experiences and pedagogies. In M. Meletious-Mavrotheris, K. Mavrou & E. Paparistodemou (Eds), *Integrating touch-enabled and mobile devices into contemporary mathematics education* (pp. 193–213). IGI Global.

Attard, C., & Curry, C. (2012). Exploring the use of iPads to engage young students with mathematics. In J. Dindyal, L. Cheng & S. Ng (Eds), *Proceedings of the 35th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 75–82). MERGA.

Bakos, S. (2022). *The teacher-tool-mathematics ensemble: Before, during and after implementing* TouchTimes. [Unpublished doctoral dissertation]. Simon Fraser University.

Bakos, S. (2023). Sociocultural influences while implementing *TouchTimes*. In P. Drijvers, C. Csapodi, H. Palmér, K. Gosztonyi, & E. Kónya (Eds), *Proceedings of the Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)* (pp. 1–8). Alfréd Rényi Institute of Mathematics and ERME.

Bakos, S., Sinclair, N., Güneş, C., & Chorney, S. (2022). Making sense(s) of multiplication. In C. Fernández, S. Llinares, A. Gutiérrez & N. Planas (Eds), *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 51–58). PME.

Barad, K. (2007). *Meeting the universe halfway: Quantum physics and the entanglement of matter and meaning*. Duke University Press.

Bartolini Bussi, M. (2009). Historical artefacts, semiotic mediation and teaching proof. In G. Hanna, H. N. Jahnke & H. Pulte (Eds), *Explanation and proof in mathematics: Philosophical and educational perspectives*. Springer. doi.org/10.1007/978-1-4419-0576-5_11

Calder, N., & Murphy, C. (2018). Reshaping the learning experience through apps: Affordances. In L. Ball, P. Drijvers, S. Ladel, H.-S. Siller, M. Tabach & C. Vale (Eds), *Uses of technology in primary and secondary mathematics education: Tools, topics and trends* (pp. 145–159). Springer International Publishing AG. doi.org/10.1007/978-3-319-76575-4_8

Clark-Wilson, A. (2010). *How does a multi-representational mathematical ICT tool mediate teachers' mathematical and pedagogical knowledge concerning variance and invariance?* [Unpublished doctoral dissertation]. University of London.

Davydov, V. (1992). The psychological analysis of multiplication procedures. *Focus on Learning Problems in Mathematics, 14*(1), 3–67.

Goos, M. (2001). Sociocultural perspectives on the learning and development of mathematics teachers and teacher-education-researchers. https://www.mathunion.org/fileadmin/ICMI/files/About_ICMI/Publications_about_ICMI/ICME_11/Goos.pdf

Goos, M. (2014). Creating opportunities to learn in mathematics education: A sociocultural perspective. *Mathematics Education Research Journal*, *26*(1), 439–457. doi.org/10.1007/s13394-013-0102-7

Guin, D., Ruthven, K., & Trouche, L. (Eds) (2005). *The didactical challenge of symbolic calculators: Turning a computational device into a mathematical instrument.* Springer.

Guin, D., & Trouche, L. (1999). The complex process of converting tools into mathematical instruments. The case of calculators. *International Journal of Computers for Mathematical Learning*, *3*(3), 195–227. doi.org/10.1023/A:1009892720043

Gutiérrez, R. (2013). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education*, *44*(1), 37–68. doi.org/10.5951/jresematheduc.44.1.0037

Haspekian, M. (2005). An "instrumental approach" to study the integration of a computer tool into mathematics teaching: The case of spreadsheets. *International Journal of Computers for Mathematical Learning*, *10*(2), 109–141. doi.org/10.1007/s10758-005-0395-z

Haspekian, M. (2007). Evolution des usages du tableur. In J.-B. Lagrange (Ed.), Genèses d'usages professionnels des technologies chez les enseignants (pp. 29–32).

Haspekian, M. (2014). Teachers' instrumental geneses when integrating spreadsheet software. In A. Clark-Wilson, O. Robutti & N. Sinclair (Eds), *The mathematics teacher in the digital era: An international perspective on technology focused professional development* (pp. 241–275). Springer. doi.org/10.1007/978-94-007-4638-1_11

Haspekian, M. (2017). Computer science in mathematics' new curricula at primary school: New tools, new teaching practices? In G. Aldon & J. Trgalová (Eds), *Proceedings of the 13th international conference on technology in mathematics teaching* (pp. 23–31). ICTMT.

Haspekian, M., & Gélis, J.-M. (2021). Informatique, Scratch et robots: De Nouvelles pratiques enseignantes en mathématiques? *Sciences et technologies de l'information et de la communication pour l'éducation et la formation*, *28*(1.1), 1–37.

Hoyles, C., Noss, R., & Kent, P. (2004). On the integration of digital technologies into mathematics classrooms. *International Journal of Computers for Mathematical Learning*, *9*(3), 309–326. doi.org/10.1007/s10758-004-3469-4

Jackiw, N., & Sinclair, N. (2019). TouchTimes [iPad application software]. *Tangible Mathematics Group*, Simon Fraser University. (https://apps.apple.com/ca/app/touchtimes/id1469862750)

Larkin, K., & Milford, T. (2018). Mathematics apps–stormy with the weather clearing: Using cluster analysis to enhance app use in mathematics classrooms. In N. Calder, K. Larkin & N. Sinclair (Eds), *Using mobile technologies in the teaching and learning of mathematics* (pp. 11–30). Springer. doi.org/10.1007/978-3-319-90179-4_2

Lerman, S. (1994). Changing focus in the mathematics classroom. In S. Lerman (Ed.), *Cultural perspectives on the mathematics classroom* (pp. 191–213). Kluwer Academic Publishers. doi.org/10.1007/978-94-017-1199-9_12

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 19–44). Ablex.

MacLure, M. (2010). The offence of theory. *Journal of Educational Policy*, *25*(2), 277–286. doi: 10.1080/02680930903462316

MacLure, M. (2013). The wonder of data. *Cultural Studies ↔ Critical Methodologies*, *13*(4), 228–232. doi.org/10.1177/1532708613487863

Maschietto, M., & Trouche, L. (2010). Mathematics learning and tools from theoretical, historical and practical points of view: The productive notion of mathematics laboratories. *ZDM: The International Journal on Mathematics Education*, *42*(1), 33–47. doi.org/10.1007/s11858-009-0215-3

Monaghan, J. (2004). Teachers' activities in technology-based mathematics lessons. *International Journal of Computers for Mathematical Learning*, *9*(3), 327–357. doi.org/10.1007/s10758-004-3467-6

Pierce, R., & Ball, L. (2009). Perceptions that may affect teachers' intention to use technology in secondary mathematics class. *Educational Studies in Mathematics*, *71*(3), 299–317. doi.org/10.1007/s10649-008-9177-6

Rabardel, P. (2002). People and technology: A cognitive approach to contemporary instruments (H. Wood, Trans.; 2nd ed.). Université Paris 8. (Original work published 1995).

Robert, A., & Rogalski, J. (2002). Le système complexe et cohérent des pratiques des enseignants de mathématiques: Une double approche. *Canadian Journal of Science, Mathematics and Technology Education*, *2*(4), 505–528. doi.org/10.1080/14926150209556538

Ruthven, K. (2014). Frameworks for analysing the expertise that underpins successful integration of digital technologies into everyday teaching practice. In A. Clark-Wilson, O. Robutti & N. Sinclair (Eds), *The mathematics teacher in the digital era: An international perspective on technology focused professional development* (pp. 373–393). Springer. doi.org/10.1007/978-94-007-4638-1_16

Schleppegrell, M. (2010). Language in mathematics teaching and learning. In J. Moschkovich (Ed.), *Language and mathematics education: Multiple perspectives and directions for research* (pp. 73–112). Information Age Publishing.

Sedaghatjou, M., & Rodney, S. (2018). Collaborative Engagement Through Mobile Technology in Mathematics Learning. In N. Calder, K. Larkin & N. Sinclair (Eds), *Using mobile technologies in the teaching and learning of mathematics* (pp. 113–129). Springer. doi.org/10.1007/978-3-319-90179-4_7

Siemon, D., Breed, M., & Virgona, J. (2005). From additive to multiplicative thinking: The big challenge of the middle years. In J. Mousley, L. Bragg & C. Campbell (Eds), *Proceedings of the 42nd Conference of the Mathematical Association of Victoria* (pp. 278–286). The Mathematical Association of Victoria.

Sinclair, N., & Baccaglini-Frank, A. (2015). Digital technologies in the early primary school classroom. In L. English & D. Kirshner (Eds), *Handbook of international research in mathematics education* (3rd edn, pp. 662–686). Taylor Francis. doi.org/10.4324/9780203448946

Sinclair, N., Chorney, S., Güneş, C., & Bakos, S. (2020). Disruptions in meanings: Teachers' experiences of multiplication in *TouchTimes*. *ZDM Mathematics Education*, *52*(7), 1471–1482. doi.org/10.1007/s11858-020-01163-9

Sinclair, N., & Coles, A. (2017). Returning to ordinality in early number sense: Neurological, technological and pedagogical considerations. In E. Faggiano, F. Ferrara & A. Montone (Eds), Innovation and Technology Enhancing Mathematics Education (pp. 39–58). Springer International Publishing. doi.org/10.1007/978-3-319-61488-5_3

Sinclair, N., & Jackiw, N. (2011). *TouchCounts* [iPad application]. Tangible Mathematics Group, Simon Fraser University. (apps.apple.com/ca/app/touchcounts/id897302197?mt=8)

Thomas, M., & Palmer, J. (2014). Teaching with digital technology: Obstacles and opportunities. In A. Clark-Wilson, O. Robutti & N. Sinclair (Eds), *The mathematics teacher in the digital era: An international perspective on technology focused professional development* (pp. 71–89). Springer. doi.org/10.1007/978-94-007-4638-1_4

Trigueros, M., Lozano, M., & Sandoval, I. (2014). Integrating technology in the primary school mathematics classroom: The role of the teacher. In A. Clark-Wilson, O. Robutti & N. Sinclair (Eds), *The mathematics teacher in the digital era: An international perspective on technology focused professional development* (pp. 111–138). Springer. doi.org/10.1007/978-94-007-4638-1_6

Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematics Learning*, *9*(3), 281–307. doi.org/10.1007/s10758-004-3468-5

Valsiner, J. (1997). *Culture and the development of children's action: A theory of human development*. (2nd ed.) John Wiley & Sons.

Vérillon, P., & Rabardel, P. (1995). Cognition and artifacts. *European Journal of Psychology of Education*, *10*(1), 77–101. doi.org/10.1007/BF03172796

Vygotsky, L. (1978). *Mind in society: The development of higher psychological processes*. (M. Cole, V. John-Steiner, S. Scribner & E. Souberman, Eds). Harvard University Press.

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