# Probability from Fourth to Sixth Grade 

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#### Abstract

The appeal of probability theory in everyday activities has resulted in its incorporation into the curricula of most countries. However, confusion persists within the international scientific community regarding the suitable age for introducing students to probability concepts. Several researchers argue that children at a young age are unable to understand the concepts of probability, while other researchers believe that with appropriate teaching interventions, children can comprehend probabilistic concepts. This study aims to address the controversial issue by conducting a study involving two groups of students. In the first group, the researchers provided a short probability teaching session and subsequently asked the children to complete a worksheet related to the topic. In the second group, students complete the worksheet without any prior teaching. The study results show that teaching has a positive effect on the development of students' knowledge and perceptions regarding probabilities. As a consequence, new opportunities and requirements emerge in the field of probability education.


Keywords: Probability; Teaching; Primary school.

## Introduction

Mathematics has played an important role in the advancement of humanity, which justifies its inclusion in the study of history (Gürbüz, 2010). In fact, possessing mathematical knowledge is considered a fundamental formal qualification, as it fosters the development of higher-level abilities such as abstract thinking, reasoning, justification, analysis, generalization, and more (Gürbüz, 2010). Currently, these skills rank high on the list of qualifications demanded in the job market. Thus, there is a continual tendency to improve the teaching of mathematics, aiming to align with the needs of contemporary individuals and their needs.

Probability is a relatively new branch of mathematics, but it quickly found assimilation through its applications and procedures in various sciences and everyday life (Batanero et al., 2004). For example, probabilities are extensively applied in economics, medicine, insurance companies, industry, and generally in any situation that exhibits uncertainty (English \& Watson, 2016). The importance of this field of mathematics is underscored by two key factors. Firstly, there is a persistent tendency to modify the curricula of various countries to incorporate probabilities (Gürbüz, 2010). Secondly, the theory of probabilities is closely linked to critical perception and decision theory concepts (Batanero et al., 2004; Zorzos \& Avgerinos, 2021), further emphasizing its importance in practical applications. As a result, probability theory serves as a crucial tool for cultivating skills and enhancing reasoning abilities (Gürbüz, 2010). In fact, many claim that the ability to estimate probabilities of potential outcomes is indispensable for actively engaging in democratic decision-making (English \& Watson, 2016).

Thus, teaching probability becomes imperative and probability literacy is required for life beyond the confines of school (English \& Watson, 2016). For this reason, many countries include probability in their curriculum. However, there are variations and inconsistencies regarding the age at which students should be introduced to this theory (HodnikČadež \& Škrbec, 2011). Some proponents argue that children are better prepared to grasp probabilistic concepts during secondary school, while others believe that introducing early probabilistic concepts in primary school would yield more favorable teaching outcomes (Tsakiridou \& Vavyla, 2015).

The international literature primarily focuses on sixth and seventh-grade students, along with research examining students' perceptions and misconceptions about probabilities (HodnikČadež \& Škrbec, 2011). Also, several researchers have sporadically addressed the early probabilistic concepts suitable for elementary school children (Tsakiridou \& Vavyla, 2015). However, a notable gap exists in research specifically addressing the appropriate age to introduce probability teaching to students. Therefore, this research aims to fill this gap in the international literature by primarily focusing on students abilities from the fourth to sixth-grade to comprehend probabilistic concepts and procedures.

The present research is not another examination of elementary school students' perceptions of probability, but rather an investigation of their understanding of probability concepts. Thus, this specific research aims to address the following research questions: a) How well can students from fourth to sixth grade, comprehend basic probability concepts after a brief educational intervention? and b) Does an educational intervention help young learners in effectively solving and reasoning probabilistic activities?

This work defines any mathematically acceptable processes and problems subject to probabilities as probabilistic problems and probabilistic processes. Moreover, it is essential to emphasize that basic probabilistic concepts encompass the fundamental introductory concepts of probability theory, including "chance experiment," "event," "'certain /impossible event," "probability," "independent events," and "repetitions of an experiment." The study commences by elucidating the nature of probabilistic knowledge. It subsequently explores the prevailing trends within the international community concerning the appropriate age for probability education. Furthermore, a dedicated section discusses the aspirations of the international scientific community regarding the teaching of probabilities in primary school. Following that, the paper endeavors to summarize the responses of young students to probabilistic procedures based on international surveys. Subsequently, the study presents the research methodology chapter, offering a comprehensive description of the method and steps followed during the research. Finally, the study concludes with the discussion and conclusions chapters, where the reader can find the overall conclusions drawn from the research.

## The Nature of Probabilistic Knowledge

Probability, as a branch of mathematics, originated from human curiosity about gambling (Gürbüz, 2010). Yet, in a remarkably brief span, the theory of probability gained resonance in many fields, including science, economics,
insurance companies, physics, technology, and sports (Gürbüz, 2010). The applications of this theory in diverse fields have indeed opened new horizons and facilitated the development of various sciences.

In probability theory, several researchers argue that probabilistic reasoning diverges from logical reasoning (Batanero et al., 2004). This distinction stems from the fact that logical reasoning is predicated on being either true or false, whereas probabilistic reasoning is expressed based on the degree of certainty, presenting a more analytical situation. Precisely, as an illustration, a logical sentence such as "tomorrow it will rain" can be classified as either true or false under logical reasoning. However, under probabilistic reasoning, the same sentence could be expressed as "there is an $80 \%$ chance it will rain tomorrow," which lies in a realm of uncertainty, where it is neither necessarily false nor true. Indeed, the theory of probability grapples with certain contentious ideas and inherently challenging concepts, with the concept of "randomness" being a typical example (Batanero et al., 2004). This characteristic defines probability as one of the mathematical domains most affected by intuitive perceptions (Gallistel et al., 2014).

Indeed, apart from the classical view that intuitive perceptions invariably influence new knowledge, probability theory does not demand extensive prior knowledge. This characteristic may lead individuals to believe that they can explain a probabilistic phenomenon using logical reasoning (Gallistel et al., 2014). Exactly, experience and intuition often prove insufficient when dealing with problems that demand knowledge (Batanero et al., 2004). The study of probability highlights the difference between the intuition and the actual measurement of the expectation of an outcome's occurrence in a given situation (Batanero et al., 2004). For instance, if the first four tosses of a coin result in consecutive heads, the probability of the next toss remains unaffected. However, people often rely on intuitive assumptions about the outcome of the next toss (Batanero et al., 2004). Such examples lead to the conclusion that the teaching of probability theory benefits considerably from the use of experiments (Gürbüz, 2010), diagrams such as tree diagrams. By employing these methods, children can better comprehend probabilistic problems and develop an intuitive insight into the benefits of the theory (Batanero et al., 2004).

## Age and Probability Teaching

In recent years, the pedagogy of probability has garnered important attention, with many countries making efforts to incorporate the theory into their curricula (Tsakiridou \& Vavyla, 2015). The international scientific community agrees and applauds such an undertaking, however, there are differing opinions about the age at which probabilistic concepts should be taught (Tsakiridou \& Vavyla, 2015). This story has its roots in the 1960s-1970s when several studies were conducted. In 1975, Piaget and Inhelder were the first researchers to systematically examine elementary school students' understanding of probabilities. The findings of Piaget and Inhelder's study revealed that probabilistic thinking is beyond the abilities of young students. Threlfall (2004) also agrees with this finding, and he advocates for teaching the theory of probability to students over the age of 16 . Since Threlfall's time, several studies have emerged that challenge the findings of Piaget and Inhelder, supporting the introduction of probabilistic concepts from elementary school (Andrew, 2009; English \& Watson, 2016; Sharma, 2015; Tsakiridou \& Vavyla, 2015; Zorluoğlu et al., 2019).

Fischbein, in 1975, asserted that students' perception of probability should be well-developed by the time they reach middle school age. After all, in research conducted by Zorzos and Avgerinos (2021), they demonstrated several benefits in the critical perception of young children who learn probabilities from elementary school. Additionally, Williams and Nisbet (2014) showed in their work that teaching probability can lead to improvements in students' attitudes and beliefs about mathematics in the short term. As mentioned earlier, Threlfall (2004) agreed with Piaget and Inhelder's findings. However, he acknowledged the importance of introducing early stages of probabilistic concepts to students from a young age, ensuring a gradual and thorough understanding of the idea of probability. In his work, Threlfall envisaged that with the appropriate conditions, young students could potentially cope with probabilistic problems. Sharma (2015) highlighted the importance of suitable learning environments for teaching probability in primary school. English and Watson (2016) reinforced this, showing that everyday activities connecting probability to real-life situations improve students' understanding. Zorluoğlu et al. (2019) extended the findings of Sharma et al., showing that appropriate educational interventions can enable students with non-typical development, particularly those with visual impairments, to understand and think probabilistically. Furthermore, Zorzos and Avgerinos (2023) demonstrated that children aged 8 to 12 years, when provided with appropriate visual representations, can effectively solve probabilistic problems and cope with probabilistic procedures, even without prior formal instruction in probabilities. As a result, many developing countries have incorporated probability education in primary school curricula (Zobenica et al., 2016). Moreover, in Greece, the Netherlands, Hungary, Italy, and the United Kingdom, the concept of probability is introduced in primary school at the age of 10 to 11 (Threlfall, 2004).

## Ambitions for teaching probability in the elementary school

The preceding paragraph underscores the emerging trend in probability education, advocating for its introduction in primary schools. Consequently, it is reasonable to question the knowledge that students in this age group are expected to acquire. According to the UK National Curriculum, a key aim of teaching probability in primary school is to develop early ideas about probability, the expected frequency of an outcome, and all possible outcomes of an event (Threlfall, 2004). Specifically, the curriculum seeks to provide a general understanding of the concept of probability without involving calculations of the probability of an event (Threlfall, 2004).

Sharma (2015) stresses the significance of introducing probabilistic concepts early, providing students with sufficient time and attention to build a solid mathematical foundation. This early exposure fosters a valuable perspective for future learning of probability theory. Sharma (2015) and Threlfall (2004) do not anticipate students providing mathematically complex responses to probabilistic activities. Such responses would necessitate advanced probabilistic knowledge, often not possessed by primary school teachers (Gómez-Torres et al., 2016).

In 2015, Tsakiridou and Vavyla surveyed 404 students in second to sixth grades of primary school. The results revealed that most students could identify probabilistic events and categorize them based on their likelihood of happening. Nonetheless, a noticeable contrast in skills emerged between small and large class sizes. In a subsequent study, English and Watson (2016) examined fourth-grade students' comprehension and aptitude in relating
experimental relative frequencies to theoretical probabilities. English and Watson's findings revealed that students effectively linked simulated computer test relative frequencies with theoretical probability and proficiently constructed and interpreted theoretical probability models. This signified a heightened grasp of theoretical probability among students (English \& Watson, 2016). Zorzos and Avgerinos (2023) conducted noteworthy research demonstrating the capability of 8 to 12-year-old students to engage in probabilistic thinking and solve exercises with sound reasoning, despite the absence of strict mathematical precision.

## Young Learners' Response to Probabilistic Procedures

Effective instruction in probability for young children necessitates suitable activities and customized teaching methods to ensure comprehension (Gürbüz, 2010). However, Threlfall (2004) suggests avoiding excessive demands on young students. Educators aiming to teach probability at the elementary level should initially address students' preexisting notions and superstitious beliefs about luck (Williams \& Nisbet, 2014).

Numerous studies have been conducted at various intervals to highlight the perceptions that students acquire from probability-focused teaching interventions. For instance, in the study by HodnikČadež and Škrbec (2011), a cohort of 623 students from the first three grades of primary education took part and engaged in solving probabilistic tasks, without any instructional intervention. The research findings revealed a discrepancy from the researchers' initial expectations, as only half of the students demonstrated the ability to categorize events based on their probabilities and differentiate between certain possible and impossible probabilities. Notably, an important outcome of the study was that teachers anticipated lower levels of performance from their students. In the study conducted by English and Watson (2016) involving fourth-grade students, a significant proportion of the participants demonstrated a grasp of the goals and challenges posed by probabilistic activities. This trend aligns with the findings of Tsakiridou and Vavyla (2015), who worked with children spanning second to sixth grades; however, notable success seemed more pronounced in the older grade levels. Zobenica et al. (2016) assert, based on their research, that elementary school children can engage with fundamental probabilistic knowledge and processes. Their study further highlights students' intuitive grasp of probabilities, suggesting that even kindergarteners possess the capacity to comprehend basic probabilistic concepts. While the concept of probability is often regarded as intricate, posing challenges for both children and adults (Threlfall, 2004), literature suggests that elementary school students exhibit promising adaptability to foundational probability concepts (English \& Watson, 2016).

## Method

This study was conducted on the island of Rhodes, involving a research sample of 177 students from 2 primary schools on the island. Specifically, the experimental group, referred to as school A, comprised 90 students from the first primary school. In detail, the experimental group from school A included 36 students from the fourth grade ( 2 sections), 37 students from the fifth grade ( 2 sections), and 17 students from the sixth grade ( 1 section). Among these, 44 were boys and 46 were girls. The control group, referred to as school B, consisted of the remaining 87 students from the second primary school who participated in the research. The control group, school B, comprised 29 students from the fourth grade ( 2 sections), 30 students from the fifth grade ( 2 sections), and 28 students from the sixth grade
( 2 sections). In terms of gender distribution, there were 51 boys and 36 girls in this group. The selection of the specific classes was based on two factors. Firstly, existing literature demonstrates a positive view on teaching probabilities to advanced primary classes. Secondly, the decision was influenced by the fact that children attending the first three primary classes lack experience in both problem-solving and reading complex exercises. Consequently, these classes might struggle to independently engage with the activities.

This study follows a quantitative research approach, involving the collection of data through a questionnaire and subsequent descriptive analysis using the SPSS statistical program. This questionnaire, designed as a worksheet, was distributed directly to the sample by the researchers. To ensure the validity of this data collection method, the content of the activities was evaluated by peer teachers and professors, aligning conceptually with the study's objectives. The questionnaire's reliability was assessed independently in both the control and experimental groups using Cronbach's alpha, a measure of internal consistency. The obtained results indicate satisfactory reliability in both groups, with a Cronbach's alpha coefficient of 0.731 for the experimental group and 0.784 for the control group.

The researchers utilized a convenience sampling approach within two school units to which they had access. Participation in the study was voluntary for the students. In accordance with Greek regulations, obtaining approval from the University's research ethics committee was not necessary since educational research can be conducted in schools by pedagogical university institutions, subject to the consent of class teachers and directors. The researcher's role as an educator played a significant part in obtaining approval for the research from the two school principals and teachers. The collaborative atmosphere and shared commitment to enhancing pedagogical approaches fostered a positive response from the teachers and school directors, who warmly supported the researcher in their endeavor. The entire investigation spanned approximately two weeks. For each section, the researcher allocated one teaching hour (equivalent to 45 minutes). In the classes of school, A , the researcher effectively utilized the entire teaching duration. This was divided into two segments: the initial 25 minutes were dedicated to a teaching intervention focused on probabilities, followed by the subsequent 20 minutes during which the students were provided with a worksheet (questionnaire). In school B, the methodology differed slightly. The researcher allocated the initial 20 minutes of the class to distributing the questionnaire without preceding probability education. For subsequent 25 minutes were used for researcher-led discussions with the students. This approach presented challenges, as engaging children with unfamiliar concepts on the worksheet created a tougher and potentially stressful classroom dynamic. Nevertheless, the researcher adeptly alleviated concerns by prefacing the session. He emphasized that the worksheet was not an evaluative test and would not impact their grades. Additionally, he assured them that their names were not required. His words then fostered a sense of responsibility, highlighting the significance of their involvement in the study and the potential benefits it could bring to future generations. As a result, the children approached the assessment sheet with enthusiasm, actively engaging with its content concerning probabilistic concepts and processes. The timing and day were thoughtfully coordinated between the researcher and the class teacher, ensuring a suitable slot that didn't disrupt the teacher's lesson plans.

The researcher's brief educational intervention within the experimental group encompassed interactive teaching, inclusive class discussions, and a practical experiment. Worth noting is that this intervention targeted particular probabilistic concepts known to be comprehensible for primary school students based on existing literature. The intervention focused on key probabilistic concepts such as "chance experiment," "event," 'certain/impossible event," '"probability," "independent event,'" and 'repetitions of an experiment." The research follows the traditional Probability approach, where a numerical value between 0 and 1 represents the likelihood of a specific outcome occurring (Batanero et al., 2016).

The teaching session proceeded as follows: The researcher initiated a conversation with the students, introducing concepts like "probability," "probable," and "unlikely," and highlighted their relevance in everyday scenarios. The students were informed that probability theory addresses situations characterized by unpredictability, where outcomes cannot be foreseen based on given conditions (Ross, 2023). The researcher introduced the term "chance experiments" to describe these scenarios, while referring to the desired outcomes as "events" (Ross, 2023). To illustrate, the researcher used a dice as a tangible example to engage the students in a discussion. Through this interactive approach, the researcher aimed to assess the students' beliefs and expectations concerning the possible outcomes when rolling the dice. Together, the researcher and the students reached a consensus that rolling a dice represents a chance experiment, and an illustrative "event" within this experiment might involve outcomes where the resulting number is less than 4 . The researcher proceeded to record the possible outcomes in a table, omitting the term "sample space" due to its perceived complexity for the students. Subsequently, the experiment was carried out with 15 repetitions by the researcher, and the outcomes were carefully recorded and organized into a table. The ensuing discussion revolved around the attained results, probing whether they aligned with the students' expectations, their rationale, and overall coherence. Following the comprehensive discussion, additional instances of different scenarios were presented, aiding the students in comprehending the nature of probability as an expression of anticipated outcomes rather than a means of prediction. This approach facilitated the resolution of various queries and misconceptions that the students held. Drawing upon these insights, the teaching process culminated in the traditional definition of probability, where it is derived from the ratio of favorable outcomes in a chance experiment to the entire set of potential outcomes (English \& Watson, 2016). Consequently, the probability associated with the discussed scenarios was conveyed to the students in the form of numerical fractions. The educational intervention was intentionally designed to be flexible and openended, allowing for active participation and spontaneous discussions among the children. The researcher aimed to foster a more dynamic engagement, enabling examples to emerge from the experiment or the students' own experiences. The procedural outline of the educational intervention is illustrated in the accompanying image.

Figure 1

## Course of Educational Intervention



The research questionnaire, referred to as a "worksheet" for the students, was meticulously designed by the researchers to align with the study's objectives. The questionnaire was crafted with the intention of incorporating relatable reallife scenarios for the students. This approach aimed to imbue the questions with practical significance, utilize uncomplicated language, and captivate the students' attention effectively. Moreover, the researchers were deliberate in excluding additional mathematical topics to ensure a focused exploration of probabilistic thinking. The questionnaire was uniformly distributed among all classes in both schools, with both the researcher and the class teacher present during distribution. This presence was aimed at ensuring the students' psychological ease and reducing the likelihood of haphazardly answered questions, contributing to a more reliable dataset. Firstly, students wrote their grade on the sheet. Then answered four multiple choice questions related to "chance experiment," "probability," "event," and "Certain event." The possible answer choices for these questions were derived from an earlier pilot study involving 32 students, where they were presented as open-ended questions. The questions were designed to introduce the concept without relying on strict mathematical definitions. Instead, simple language was used to help students grasp how these concepts apply to everyday thinking, eliminating the need for detailed explanations. Following this, four activities were presented, as depicted below:

## Exercise 1

A box of cereal gives a card or sticker of the national basketball team as a gift.
a) What do you think you'll find in the box?
b) If we find the card in the first box, then in the second box we will find the sticker?

## Exercise 2

We roll a die which has 4 red sides and 2 white sides and record the result of the upper side. We repeat this experiment a second time.
a) What do you think will be the result of the first roll?
b) What do you think will be the result of the second roll?
c) Which color comes up most often after 16 rolls?

## Exercise 3

George and Anna are playing a board game and decide to flip a coin to see who will play first. Is this decision fair to both? Explain your answer.

## Exercise 4

We have rolled a die three times. We brought sixes in all three. We plan to drop it one more time. (Mark the sentences as True or False)
a) It is impossible to bring six again.
b) We are lucky and we will bring six.
c) After rolling four times, the probability is 1 from 4.
d) The probability is $\frac{1}{6}$.
e) The probability is $\frac{1}{2}$ to bring six and $\frac{1}{2}$ not to bring.

The initial activity aimed to assess students' comprehension of equally likely events and their grasp of probability during the second trial of the experiment. In the subsequent activity, the focus shifted to students' recognition of nonequivalent contingencies and their grasp of how probability impacts multiple repetitions of the experiment. The third activity involved another assessment of equal probability, but this time through a real-life scenario unrelated to traditional probability exercises. Students were required to provide justifications for their answers. Only responses accompanied by correct justifications, whether strictly mathematical or in approximate wording of the classical probability definition, were considered correct answers. The last activity serves a dual purpose: it assesses students' grasp of the classical probability definition while also probing for potential misconceptions arising from the repetition of the experiment, their intuitive perceptions, and their reliance on the notion of "luck".

The questionnaire's design and exercise selection aimed to capture the students' attention, and the researchers were prepared for numerous unanswered questions from both the experimental and control groups. This anticipation arises from the fact that probabilities constitute a mathematical concept not yet covered in their curriculum.

## Results

The research findings are displayed in 11 tables. The tables provide the frequencies of the answer results accompanied by their respective percentages in parentheses. The arrangement of the results mirrors the sequence of the questions
as presented in the questionnaire - worksheet. It is important to highlight that in the presentation of the results, the authors also provide the correct answers when needed, aiming to assist readers who may not be familiar with the subject matter.

Table 1
Questions About What Is Chance Experiment

| What is "chance experiment"? | Control Team |  |  | Experimental Team |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Grade | 4 | 5 | 6 | 4 | 5 | 6 |
| An experiment conducted at random. | $\begin{gathered} 8 \\ (27.6 \%) \end{gathered}$ | $\begin{gathered} 10 \\ (33.3 \%) \end{gathered}$ | $\begin{gathered} 7 \\ (25 \%) \end{gathered}$ | $\begin{gathered} 13 \\ (36.1 \%) \end{gathered}$ | $\begin{gathered} 11 \\ (29.7 \%) \end{gathered}$ | $\begin{gathered} 0 \\ (0 \%) \end{gathered}$ |
| An experiment that occurs by accident. | $\begin{gathered} 6 \\ (20.7 \%) \end{gathered}$ | $\begin{gathered} 6 \\ (20 \%) \end{gathered}$ | $\begin{gathered} 1 \\ (3.7 \%) \end{gathered}$ | $\begin{gathered} 1 \\ (2.8 \%) \end{gathered}$ | $\begin{gathered} 4 \\ (10.8 \%) \end{gathered}$ | $\begin{gathered} 2 \\ (11.8 \%) \end{gathered}$ |
| An experiment that we cannot predict the result that will come at a particular time. | $\begin{gathered} 13 \\ (44.8 \%) \end{gathered}$ | $\begin{gathered} 11 \\ (\mathbf{3 6 . 7 \%}) \end{gathered}$ | $\begin{gathered} 18 \\ (64.3 \%) \end{gathered}$ | $\begin{gathered} 14 \\ (38.9 \%) \end{gathered}$ | $\begin{gathered} 19 \\ (51.4 \%) \end{gathered}$ | $\begin{gathered} 5 \\ (29.4 \%) \end{gathered}$ |
| Blank Answer | $\begin{gathered} 2 \\ (6.9 \%) \end{gathered}$ | $\begin{gathered} 3 \\ (10 \%) \end{gathered}$ | $\begin{gathered} 2 \\ (7.1 \%) \end{gathered}$ | $\begin{gathered} 8 \\ (22.2 \%) \end{gathered}$ | $\begin{gathered} 3 \\ (8.1 \%) \end{gathered}$ | $\begin{gathered} 10 \\ (58.8 \%) \end{gathered}$ |
| Total | $\begin{gathered} 29 \\ (100 \%) \end{gathered}$ | $\begin{gathered} 30 \\ (100 \%) \end{gathered}$ | $\begin{gathered} 28 \\ (100 \%) \end{gathered}$ | $\begin{gathered} 36 \\ (100 \%) \end{gathered}$ | $\begin{gathered} 37 \\ (100 \%) \end{gathered}$ | 17 $(100 \%)$ |

Table 1 displays the students' responses regarding the concept of "chance experiment." The accurate response for this question pertains to an experiment of chance being defined as one in which the outcome cannot be predicted at a specific instance. The table indicates noticeable distinctions between the percentages of accurate responses in the experimental and control groups for the fourth and sixth grades. In the fourth grade, a minor difference exists, with the control group displaying a slightly higher rate of correct answers. Conversely, the sixth-grade control group demonstrated a substantially improved response rate compared to the experimental group. Conversely, the fifth-grade experimental group exhibited a marked improvement in their comprehension of the question. Of course, the difference in the percentages of students' blank answers is also noteworthy. Particularly, in the fourth and sixth-grade classes, substantial disparities exist, with the experimental group displaying notably more blank responses compared to their counterparts in the control group.

Table 2
Questions About What Is Event

| Which of the following do you think is the "event"? | Control Team |  |  | Experimental Team |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Grade | 4 | 5 | 6 | 4 | 5 | 6 |
| A possible outcome resulting from running the experiment. | $\begin{gathered} 7 \\ (24.1 \%) \end{gathered}$ | $\begin{gathered} 6 \\ (20 \%) \end{gathered}$ | $\begin{gathered} 6 \\ (21.4 \%) \end{gathered}$ | $\begin{gathered} 6 \\ (16.7 \%) \end{gathered}$ | $\begin{gathered} 6 \\ (16.2 \%) \end{gathered}$ | $\begin{gathered} 2 \\ (11.8 \%) \end{gathered}$ |
| A situation that can happen under conditions. | $\begin{gathered} 5 \\ (17.2 \%) \end{gathered}$ | 7 $(23.3 \%)$ | $\begin{gathered} 5 \\ (17.9 \%) \end{gathered}$ | $\begin{gathered} 5 \\ (13.9 \%) \end{gathered}$ | $\begin{gathered} 7 \\ (18.9 \%) \end{gathered}$ | $\begin{gathered} 2 \\ (11.8 \%) \end{gathered}$ |
| A situation that may occur in the future. | $\begin{gathered} 11 \\ (37.9 \%) \end{gathered}$ | $\begin{gathered} 10 \\ (33.3 \%) \end{gathered}$ | $\begin{gathered} 10 \\ (35.7 \%) \end{gathered}$ | $\begin{gathered} 15 \\ (41,6 \%) \end{gathered}$ | $\begin{gathered} 17 \\ (46 \%) \end{gathered}$ | $\begin{gathered} 6 \\ (35.2 \%) \end{gathered}$ |
| It has nothing to do with probabilities. | $\begin{gathered} 2 \\ (6.9 \%) \end{gathered}$ | $\begin{gathered} 3 \\ (10 \%) \end{gathered}$ | $\begin{gathered} 5 \\ (17.9 \%) \end{gathered}$ | $\begin{gathered} 6 \\ (16.7 \%) \end{gathered}$ | $\begin{gathered} 3 \\ (8.1 \%) \end{gathered}$ | $\begin{gathered} 1 \\ (5.9 \%) \end{gathered}$ |
| Blank Answer | $\begin{gathered} 4 \\ (13.8 \%) \end{gathered}$ | 4 $(13.3 \%)$ | $\begin{gathered} 2 \\ (7.1 \%) \end{gathered}$ | $\begin{gathered} 4 \\ (11.1 \%) \end{gathered}$ | $\begin{gathered} 4 \\ (10.8 \%) \end{gathered}$ | $\begin{gathered} 6 \\ (35.3 \%) \end{gathered}$ |
| Total | $\begin{gathered} 29 \\ (100 \%) \end{gathered}$ | 30 $(100 \%)$ | $\begin{gathered} 28 \\ (100 \%) \end{gathered}$ | $\begin{gathered} 36 \\ (100 \%) \end{gathered}$ | $\begin{gathered} 37 \\ (100 \%) \end{gathered}$ | 17 $(100 \%)$ |

Table 2 illustrates the participants' answers concerning the notion of "event." The accurate response to this inquiry was that an "event" signifies a potential occurrence in the future. Hence, it is evident that the experimental group demonstrated a superior comprehension of the given concept compared to the control group, with the exception of the sixth grade, where the variance is marginal in relation to the control group. Notably, the sixth grade of the experimental group also displayed a significant proportion of blank responses in this table.

Table 3
Questions About Certain Event

| When do we use the phrase | Control Team |  |  | Experimental Team |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| "'certain event'? |  |  |  |  |  |
| Grade |  |  |  |  |  |

Table 3 presents the participants' answers concerning the understanding of the concept of a "certain event." A "certain event" is defined as an event that is guaranteed to occur. Notably, both the experimental and control groups exhibit substantial percentages of accurate responses, with marginal distinctions favoring one group over the other.

Table 4
Questions About the Word "Probability"

| Where do we use the word 'probability'? | Control Team |  |  | Experimental Team |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Grade | 4 | 5 | 6 | 4 | 5 | 6 |
| In games of chance. | $\begin{gathered} 11 \\ (37.9 \%) \end{gathered}$ | $\begin{gathered} 8 \\ (26.7 \%) \end{gathered}$ | $\begin{gathered} 6 \\ (21.4 \%) \end{gathered}$ | $\begin{gathered} 13 \\ (36.1 \%) \end{gathered}$ | $\begin{gathered} 11 \\ (29.7 \%) \end{gathered}$ | $\begin{gathered} 3 \\ (17.6 \%) \end{gathered}$ |
| On forecasts. | 2 (6.9\%) | $\begin{gathered} 4 \\ (13.3 \%) \end{gathered}$ | $\begin{gathered} 6 \\ (21.4 \%) \end{gathered}$ | 2 (5.6\%) | $\begin{gathered} 4 \\ (10.8 \%) \end{gathered}$ | $\begin{gathered} 2 \\ (11.8 \%) \end{gathered}$ |
| In luck problems. | 2 (6.9\%) | 2 $(6.7 \%)$ | $\begin{gathered} 4 \\ (14.3 \%) \end{gathered}$ | $\begin{gathered} 6 \\ (16.7 \%) \end{gathered}$ | $\begin{gathered} 5 \\ (13.5 \%) \end{gathered}$ | $\begin{gathered} 1 \\ (5.9 \%) \end{gathered}$ |
| At the casino. | $\begin{gathered} 3 \\ (10.3 \%) \end{gathered}$ | $\begin{gathered} 3 \\ (10 \%) \end{gathered}$ | $\begin{gathered} 3 \\ (10.7 \%) \end{gathered}$ | $\begin{gathered} 4 \\ (11.1 \%) \end{gathered}$ | $\begin{gathered} 4 \\ (10.8 \%) \end{gathered}$ | $\begin{gathered} 0 \\ (0 \%) \end{gathered}$ |
| In all the above. | $\begin{gathered} 5 \\ (17.2 \%) \end{gathered}$ | $\begin{gathered} 9 \\ (30 \%) \end{gathered}$ | $\begin{gathered} 8 \\ (\mathbf{2 8 . 6 \%}) \end{gathered}$ | $\begin{gathered} 7 \\ (19.4 \%) \end{gathered}$ | $\begin{gathered} 8 \\ (\mathbf{3 2 . 4 \%}) \end{gathered}$ | $\begin{gathered} 7 \\ (41.1 \%) \end{gathered}$ |
| Blank Answer | $\begin{gathered} 6 \\ (20.7 \%) \end{gathered}$ | 4 $(13.3 \%)$ | 1 (3.6\%) | $\begin{gathered} 4 \\ (11.1 \%) \end{gathered}$ | 1 (2.7\%) | $\begin{gathered} 4 \\ (23.6 \%) \end{gathered}$ |
| Total | $\begin{gathered} 29 \\ (100 \%) \end{gathered}$ | 30 $(100 \%)$ | $\begin{gathered} 28 \\ (100 \%) \end{gathered}$ | $\begin{gathered} 36 \\ (100 \%) \end{gathered}$ | 37 $(100 \%)$ | $\begin{gathered} 17 \\ (100 \%) \end{gathered}$ |

Table 4 illustrates the students' reactions regarding the utilization of the term "probability." The majority of children from the two higher grade levels, as well as both experimental and control groups, predominantly selected the option indicating that the term "probability" is applicable in all the provided scenarios. On the contrary, the majority of fourth grade students from both the experimental and control groups answered that this concept is used in games of chance. Among the different age groups within the two groups, a slightly higher response rate is observed in the experimental group.

Table 5
Exercise 1 First Question

| What do you think you'll find in the box? <br> Grade | Control Team |  |  | Experimental Team |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 4 | 5 | 6 | 4 | 5 | 6 |
| Correct answer (Card or sticker, but we do not know for sure) | $\begin{gathered} 1 \\ (3.4 \%) \end{gathered}$ | $\begin{gathered} 1 \\ (3.3 \%) \end{gathered}$ | $\begin{gathered} 2 \\ (7.1 \%) \end{gathered}$ | $\begin{gathered} 9 \\ (25 \%) \end{gathered}$ | $\begin{gathered} 7 \\ (18.9 \%) \end{gathered}$ | $\begin{gathered} 9 \\ (52.9 \%) \end{gathered}$ |
| Card | $\begin{gathered} 14 \\ (48.3 \%) \end{gathered}$ | $\begin{gathered} 17 \\ (56.7 \%) \end{gathered}$ | $\begin{gathered} 12 \\ (42.9 \%) \end{gathered}$ | $\begin{gathered} 8 \\ (22.2 \%) \end{gathered}$ | $\begin{gathered} 9 \\ (24.3 \%) \end{gathered}$ | $\begin{gathered} 3 \\ (17.6 \%) \end{gathered}$ |
| Sticker | $\begin{gathered} 6 \\ (20.7 \%) \end{gathered}$ | $\begin{gathered} 8 \\ (26.7 \%) \end{gathered}$ | $\begin{gathered} 10 \\ (35.7 \%) \end{gathered}$ | $\begin{gathered} 6 \\ (16.7 \%) \end{gathered}$ | $\begin{gathered} 15 \\ (40.5 \%) \end{gathered}$ | 2 <br> (11.8\%) |
| Other | $\begin{gathered} 6 \\ (20.7 \%) \end{gathered}$ | $\begin{gathered} 0 \\ (0 \%) \end{gathered}$ | $\begin{gathered} 2 \\ (7.1 \%) \end{gathered}$ | $\begin{gathered} 5 \\ (13.9 \%) \end{gathered}$ | $\begin{gathered} 2 \\ (5.4 \%) \end{gathered}$ | $\begin{gathered} 0 \\ (0 \%) \end{gathered}$ |
| Blank Answer | $\begin{gathered} 8 \\ (27.6 \%) \end{gathered}$ | $\begin{gathered} 4 \\ (13.3 \%) \end{gathered}$ | $\begin{gathered} 2 \\ (7.1 \%) \end{gathered}$ | $\begin{gathered} 8 \\ (22.2 \%) \end{gathered}$ | $\begin{gathered} 4 \\ (10.8 \%) \end{gathered}$ | $\begin{gathered} 3 \\ (17.6 \%) \end{gathered}$ |
| Total | $\begin{gathered} 29 \\ (100 \%) \end{gathered}$ | $\begin{gathered} 30 \\ (100 \%) \end{gathered}$ | $\begin{gathered} 28 \\ (100 \%) \end{gathered}$ | $\begin{gathered} 36 \\ (100 \%) \end{gathered}$ | 37 <br> (100\%) | 17 <br> (100\%) |

Table 5 displays the students' responses to the initial question of the first exercise. It appears that there is a noteworthy disparity in the percentages of correct answers between the two groups of students. To elaborate further, in the fourth grade of the control group, only $3.4 \%$ of students answered correctly, while in the corresponding grade of the experimental group, $25 \%$ of them provided the accurate response. Likewise, in fifth and sixth grades, a significant discrepancy in correct answers between the two groups is evident.

Table 6
Exercise 1 Second Question

| If we find the card in the first box, in the <br> second box, we will find the sticker? | Control Team |  | Experimental Team |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

Subsequently, in the second question of the same exercise, the percentages of responses from the experimental group continue to be higher. However, what stands out is the notable occurrence of the answer "no," indicating that they will not discover the sticker in the next box.

Table 7
Exercise 2 First Question

| What do you think will be the result of <br> the first roll? | Control Team |  | Experimental Team |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

Exercise 2 presents the outcomes of its initial question in Table 7. The table indicates relatively modest success rates in student responses, particularly among the two smallest participating classes. Notably, the results in the experimental group appear slightly more favorable, both in terms of affirmative answers and instances of blank responses.

Table 8
Exercise 2 Second Question

| What do you think will be the result of the |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| second roll? | Control Team

Continuing along the lines of the initial question within the same exercise, the investigation proceeds as depicted in Table 8. Examining the second roll of the dice, Table 8 illustrates an escalation in the proportions of students who anticipate a white outcome this time around.

Table 9
Exercise 2 Third Question

| Which color comes up most often after 16 <br> rolls? | Control Team |  | Experimental Team |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Grade | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| Correct Answer (We cannot know for <br> sure) | 8 | 10 | 12 | 7 | 3 | 6 |
|  | $(27.6 \%)$ | $(33.3 \%)$ | $(42.9 \%)$ | $(19.4)$ | $(8.1 \%)$ | $(35.3 \%)$ |
| White | 5 | 5 | 8 | 2 | 5 | 1 |
|  | $(17.2 \%)$ | $(16.7 \%)$ | $(28.6 \%)$ | $(5.6 \%)$ | $(13.5 \%)$ | $(5.9 \%)$ |
| Red | 1 | 1 | 3 | 0 | 0 | 9 |
|  | $(3,4 \%)$ | $(3.3 \%)$ | $(10.7 \%)$ | $(0 \%)$ | $(0 \%)$ | $(52.9 \%)$ |
| Blank Answer | 15 | 14 | 5 | 27 | 29 | 1 |
|  | $(51.7 \%)$ | $(46.7 \%)$ | $(17.9 \%)$ | $(75 \%)$ | $(78.4 \%)$ | $(5.9 \%)$ |
| Total | 29 | 30 | 28 | 36 | 37 | 17 |
|  | $(100 \%)$ | $(100 \%)$ | $(100 \%)$ | $(100 \%)$ | $(100 \%)$ | $(100 \%)$ |

Table 9 displays the outcomes of the final question within exercise 2. Notably, the control group outperforms the experimental group in this table. Additionally, it is significant to highlight the substantial proportions of unanswered responses in the first two classes of the experimental group.

During the third exercise of the evaluation sheet, justification was required for the correct answer. It was acknowledged that the justification need not adhere strictly to mathematical principles, given the young age of the students and their limited experience in this area. Thus, an acceptable justification was any written explanation that demonstrated an understanding of the equal probability of the two contingencies. As depicted in Table 10, the fourth-grade students from the control group demonstrated slightly better performance compared to their counterparts in the experimental group, whereas the reverse was observed in the two older classes. Additionally, the percentages of blank answers are noteworthy in this context.

Table 10
Exercise 3 the Fair Game

| Flipping a coin. Is this decision fair? | Control Team | Experimental Team |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Grade | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| Correct Answer | 16 | 13 | 18 | 19 | 26 | 12 |
|  | $(55.2 \%)$ | $(43.3 \%)$ | $(64.3 \%)$ | $(52.8 \%)$ | $(70.3 \%)$ | $(70.6 \%)$ |
| Wrong Answer | 3 | 3 | 7 | 6 | 4 | 4 |
|  | $(10.3 \%)$ | $(10 \%)$ | $(25 \%)$ | $(16.7 \%)$ | $(10.8 \%)$ | $(23.5 \%)$ |
| Blank Answer | 10 | 14 | 3 | 11 | 7 | 1 |
|  | $(34.5 \%)$ | $(46.7 \%)$ | $(10.7 \%)$ | $(30.6 \%)$ | $(18.9 \%)$ | $(5.9 \%)$ |
| Total | 29 | 30 | 28 | 36 | 37 | 17 |
|  | $(100 \%)$ | $(100 \%)$ | $(100 \%)$ | $(100 \%)$ | $(100 \%)$ | $(100 \%)$ |

## Table 11

Answers of Exercise 4
After rolling a dice three times and comes
all times 6. What will happen in the next Control Team
rolling?

| Grade | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| It is impossible to bring six again. | 10 | 2 | 0 | 3 | 6 | 14 |
|  | $(34.5 \%)$ | $(6.7 \%)$ | $(0 \%)$ | $(8.3 \%)$ | $(16.2 \%)$ | $(82.4 \%)$ |


| We are lucky and we will bring six. | 8 | 12 | 14 | 14 | 14 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(27.6 \%)$ | $(40 \%)$ | $(50 \%)$ | $(38.9 \%)$ | $(37.8 \%)$ | $(52.9 \%)$ |


| After rolling four times, the probability is | 4 | 6 | 5 | 8 | 9 | 7 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ from 4. | $(23.8)$ | $(20 \%)$ | $(17.9 \%)$ | $(22.2 \%)$ | $(24.3 \%)$ | $(41.2 \%)$ |

The probability is $\frac{1}{6}$.

| 5 | 6 | 7 | 4 | 4 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $(17.2 \%)$ | $(20 \%)$ | $(25 \%)$ | $(11.1 \%)$ | $(10.8 \%)$ | $(41.2 \%)$ |

The probability is $\frac{1}{2}$ to bring six and $\frac{1}{2}$ not to bring.
(3.4\%)
(3.3\%)

4
(14.3\%) (16.7\%) (10.8\%)

7
(41.2\%)

Finally, Table 11 shows the results of the fourth exercise, which comprised a True-False assessment. Notably, there were no instances of blank answers in this exercise. Therefore, the table exclusively displays the correct answers provided by the students. Specifically, in the first question, it is evident that the fourth-grade students from the control group outperformed their counterparts in the experimental group. On the contrary, in the larger classes, the students of the experimental group show a higher percentage. In the second and third questions, students in the experimental group show a higher percentage of correct results. Without, however, the individual percentages in the classes of the experimental group being greater than the corresponding classes in the control group. In the fourth question, the two smallest classes of the control group exhibited better responses than the experimental group. However, in the last question, each class of the experimental group achieved higher percentages of accurate answers compared to the corresponding classes of the control group.

## Discussions

This study aimed to both establish a standpoint and address the question of the optimal age for introducing probabilistic concepts. To achieve this, the research targeted students in the fourth, fifth, and sixth grades of primary school. This choice was guided by existing literature, which suggests that these age groups possess varying degrees of ability to comprehend probabilistic concepts (English \& Watson, 2016; Tsakiridou \& Vavyla, 2015; Zorzos \& Avgerinos, 2021). Additionally, this study sought to underscore the preparedness of students by conducting a brief pedagogical intervention on probabilities, with the intention of equipping them to tackle probabilistic challenges. In essence, the research aimed to demonstrate that children can be guided toward acquiring mathematical proficiency in probabilities through a targeted educational intervention. This focus distinguished the study from mere investigations into students' perceptions or their experiential encounters with the mathematical realm of probabilities. The findings of the study, as outlined in the preceding section, indicate that students in the experimental group, who participated in the educational intervention and the researcher's experiment, generally exhibited improved performance in probabilistic problem-solving and a deeper comprehension of probabilistic concepts.

Probability is a mathematical field that does not necessitate a specialized cognitive foundation. However, grasping probabilistic concepts can prove challenging due to the distinct nature of probabilistic reasoning when compared to logical reasoning (Batanero et al., 2004). This is also one of the main reasons for the disagreement of researchers regarding the acceptance of an age at which probabilities can be introduced. This study is in agreement with the research and curricula of the countries that support the introduction of probabilities in primary school. Certainly, given the tender age of students, there exists a consensus among researchers that the introduction of probabilistic concepts in primary education should encompass fundamental ideas and techniques (Zobenica, 2016). While this study might have ventured into slightly more intricate concepts, its outcomes illuminate fresh avenues for both students and pedagogy.

The prevalence of significant percentages of blank responses in various instances of the research findings was unsurprising, given the students' youth and their limited exposure to probabilistic concepts. Of course, in certain questions and not all age groups, blank answers appear more in the experimental group (Tables 1, 2, 4, 5, and 9),
where the children had the advantage of the teaching intervention and the discussion with the teacher and researcher about probabilistic procedures. This could be attributed to the experimental group children's fatigue, as the teaching intervention preceded the worksheet completion. After all, they are 9 to 11 years old.

In certain multiple-choice questions (Tables 1, 2 and 3), the control group occasionally outperforms the experimental group in correct responses. This could be due to a random event, or perhaps the brief teaching intervention did not allow students ample time and space to comprehend the concepts. Nonetheless, the results showcased in Tables 1, 2, and 3 are quite promising, demonstrating the majority of students in both the experimental and control groups have a positive grasp of the concepts "chance experiment," "event," and "certain fact." This finding, alongside Table 4 showing "probability" concept question results, aligns with Threlfall's (2004) advocating for early ideas about probability in primary school. In fact, this research extends Threlfall's (2004) study by showing that in the majority of the classes of both groups, the children who underwent the educational intervention have higher positive response percentages.

Tables 5 and 6 show student responses in the case of equal probability and experiment repetition. Experimental group students appear to have handled the activity's two questions quite well, understanding that the experiment's second execution is independent of the first. This reveals a positive effect of teaching probability at this age on the understanding of independent contingencies. Thus, the findings presented in the two tables align with English and Watson's (2016) and Tsakiridou and Vavyla's (2015) research, both regarding the recognition of probabilistic events and student success in probabilistic activities, with more advanced classes having higher success rates.

In Tables 7 and 8, summarizing the results of the non-equivalent contingencies questions and the experiment repetition for the second activity, the experimental group students again show a better picture in positive responses. Of course, in Table 9, housing all answers to the second exercise's third question, it appears that the percentages favor the control group. Possibly at this juncture, the number of times the researcher repeated the experiment during the teaching intervention had an effect. As Donoghue and his team (2021) explored, this could be mitigated with more teaching time potential use of some technological monitoring methods, to present the experiment to the children with multiple repetitions. This could be implemented in future research, thereby examining, and linking theoretical probability to relative frequency, hence broadening English and Watson's (2016) research. Then, in Table 10, high percentages of positive responses from both groups stand out. Indeed, a large percentage of students seem to justify their answers quite well, particularly in the two advanced classes. This once again affirms English and Watson's (2016) and Tsakiridou and Vavyla's (2015) viewpoints yet contradicts Sharma's (2015) and Threlfall's (2004) research, who argue that one should not expect well-reasoned and mathematized answers to probabilistic activities from elementary school students. In Table 11, it appears that the sixth-grade children in the experimental group exhibit high rates of understanding of the classical definition and its experiment repetitions. In younger classes, there are lower percentages, particularly student confusion when presented with the probability as a fraction. Indeed, in the last
question, it appears that the majority of the control group students believe that the possible results are either getting a 6 or not, a notion that although improves in the experimental group, still remains at low percentages.

The research aligns with and expands on the findings of HodnikČadež and Škrbec (2011), regarding the differentiation of facts and probabilities, as well as understanding certain and impossible events. In fact, by extending HodnikČadež and Škrbec's (2011) research, this research shows that with the appropriate teaching intervention, students in advanced elementary classes can understand slightly more complex concepts than what is generally supported by the international literature. Thus, the work aligns with Gürbüz (2010), who asserted that appropriate activities and adapted teaching can lead young students to understand the concept of probability. As a result of the research, along with research such as Zorzos and Avgerinos (2023), Zorluoğlu and her team (2019) and Zobenica and her colleagues (2016), it is concluded that students can cope with probabilistic processes with the appropriate teaching intervention. Evidently, the older elementary grades have more fluency and capacity to understand probabilistic concepts, yet even in the middle grade, the research looks promising for the future and speculates that with more instructional time, the results would be even better.

## Conclusion and Limitations

In summary, this study explores the understanding of some basic probabilistic concepts by fourth to sixth-grade students, through a short teaching intervention. The study results show that the teaching intervention, albeit short for the specific research, was capable of enhancing students' perceptions, knowledge and skills in probabilistic activities. Such abilities and knowledge are essential for modern individuals, leading many to argue that they should be introduced and nurtured from primary school age.

This work highlights the capacity of the students, through a small didactic intervention in probabilities, to grasp probabilistic concepts. The age at when students can learn probabilistic concepts appears to be the sixth grade; however, some early concepts and processes can be successfully introduced to fourth graders. Therefore, the results of this study, combined with the international literature, introduce new knowledge to the research community. They conclude that a suitably targeted teaching intervention can guide students to both learn complex probabilistic concepts for their age and develop abilities to handle probabilistic tasks and justify their responses. Indeed, based on the research, one could make the educated guess that by dedicating the necessary time and suitable supervisory means, the research results and the positive responses from the students in the experimental group could be further enhanced.

The results of this work could be extrapolated to other areas of science. For example, they could be useful when designing school textbooks, syllabi or even other research aimed at improving the teaching of probability and mathematics in general. They could also assist and be utilized by a researcher or teacher, serving as a survey of students' perceptions of basic probabilistic concepts (control group) and as a survey of students' knowledge aspirations following a properly designed teaching intervention. Indeed, it would be quite intriguing as a future perspective of the study to conduct the research on the same sample at different time periods, thereby highlighting the evolution of students' perceptions and ultimately the attainment of learning. Another interesting future perspective is for the
research to allocate more teaching time to the intervention and employ, in addition to the experiment, other supervisory technological means, which can deliver more repetitions in a short period of time.

It is worth noting at this point that the present study exhibited some limitations. Initially, only research available in English and Greek was studied, and it was also not feasible to study research from conferences and workshops, which are not available online. The generalizability of this study is limited as it only involves students from the Greek education system and the sample is limited to a convenience sampling. Another limitation of the study is that the research was conducted in two different samples. This, in itself, could be a future prospect for conducting the survey on the same sample with sufficient time between the administration of the worksheets. Lastly, other future perspectives of the research in question that are worth mentioning and could mitigate some limitations of the study might include the use of technology during the experiment's conduct and the allocation of more teaching hours to deliver a probability lesson, thereby allowing students time to grasp new probabilistic concepts.

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