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Investigating Pedagogies in Undergraduate Precalculus and their Relationships to Students' Attitudes Towards Mathematics and Perseverance in Problem-Solving

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Abstract: This exploratory study investigated the relationships between professors' enactments of a research-based precalculus curriculum (Pathways) and changes observed in students' attitudes towards mathematics and perseverance in problem-solving. While much research has focused on improving student achievement in undergraduate STEM courses, it is also important to investigate methods of supporting students in developing the positive dispositions and practices needed to sustain them through years of mathematics-based STEM coursework. We therefore investigated the way three professors implemented Pathways, assessing via observation their pedagogical choices across three Pathways-aligned dimensions; we also investigated the changes in attitudes and perseverance of students in these professors' classes. Our results suggest that although the Pathways precalculus curriculum may support the development of positive attitudes toward mathematics and improved perseverance in problem-solving, this potential is influenced by professors' pedagogical choices. This research helps us, and the field of undergraduate mathematics education, better understand the connections between pedagogies enacted in STEM-gateway courses and students' development of productive ways of engaging with mathematics.

Keywords: Attitudes towards mathematics; Curriculum; Perseverance in problem-solving; Precalculus; Undergraduate Mathematics Education.
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Introduction

Introductory undergraduate mathematics courses are often positioned as gatekeeper courses for STEM majors, and negative experiences in these courses are often cited as the reason students abandon their STEM majors (Bressoud et al., 2013; Seymour & Hunter, 2019). In response, researchers in undergraduate mathematics education often focus their attention on how to improve student success rates in these introductory courses, including the development of innovative curricula (e.g., Boelkins et al., 2018; Carlson et al., 2021). Precalculus may be especially in need of such attention, since the mathematical ideas emphasized in precalculus courses, including quantitative reasoning, variational and covariational reasoning, and functions, are foundational for STEM majors' success in calculus and beyond (Thompson & Carlson, 2017). While improving academic outcomes in courses like precalculus is important, researchers must also investigate ways of fostering students' development of the positive dispositions and practices needed in mathematics-based STEM coursework (Bressoud et al., 2015). For example, nurturing students' development of positive attitudes towards mathematics may help them to succeed in calculus (Sonnert et al., 2020) and to complete their STEM degree (Wu et al., 2022). Further, fostering students' perseverance in problem-solving helps them to make meaning of mathematics (Jacobson et al., 2024) and develop dispositional factors that could be productive for STEM majors, such as building resiliency for overcoming setbacks and discouragements (Middleton et al., 2015). Precalculus is an apt setting for supporting students' perseverance, since the mathematical ways of thinking stressed in precalculus courses require productive struggles by the learner (Thompson & Carlson, 2017). Even with innovative curricula that support these outcomes, instructors enact curricula differently, which can transform students' opportunities to learn (Stein et al., 2007), including outcomes associated with mathematical practices and dispositions (DiNapoli, 2023; DiNapoli & Miller, 2022; Ruthven, 2011). In this paper, we report on an exploratory study on the potential connections between professors' different pedagogical enactments of a research-based precalculus curriculum and their students' changes in attitudes towards mathematics (henceforth, *attitudes*) and perseverance in problem-solving (henceforth, *perseverance*).

The *Pathways* Curriculum

The *Precalculus: Pathways to Calculus* curriculum (Carlson et al., 2021) was developed and refined based on research on the understandings and reasoning abilities students need to succeed in calculus (Carlson et al., 2002; Carlson et al., 2010). Studies have shown that *Pathways* can support students in making connections between precalculus concepts (O'Meara & Vaidya, 2021) and have linked the *Pathways* curriculum to students' improved covariational reasoning, conceptual learning of precalculus concepts, and subsequent success in calculus (Carlson et al., 2003; McNicholl et al., 2021; Moore & Carlson, 2012). *Pathways* was designed to assist precalculus instructors in surfacing and advancing student thinking through rich problem-solving experiences that help students to make mathematical connections across concepts, functional representations, and to real-world contexts (Carlson et al., 2021). Moreover, the *Pathways* authors leveraged research into how students make sense of precalculus concepts (e.g., Frank, 2017; Flores, 2018; Moore, 2010) to develop the scaffolded questioning included in their *investigations*, which are contextualized problem sets designed for students to explore in small-group problem-solving settings. Such experiences have the potential to foster positive student outcomes beyond academic performance, such as increased enjoyment of mathematics (Noyes, 2012), motivation to persist in mathematics (Peterson, 2019), and perseverance development (Barnes, 2019; DiNapoli & Miller, 2022; DiNapoli & Morales, Jr., 2021; Barnes, 2019; Morales, Jr. & DiNapoli, 2018; Schoenfeld, 1992).

Precalculus Instructional Practices and Related Student Outcomes

The connection between what happens in undergraduate precalculus classrooms and student outcomes is often studied in the context of classes conducted using instructional practices that actively engage students in problem-solving. Many of these studies characterize the efficacy of these practices in terms of students' grades, passing rates, or success in subsequent calculus courses (e.g., Collins, 2019; Goyer et al., 2020; Gruber et al., 2021; Morris et al., 2023). For example, Collins (2019) compared academic outcomes for students in precalculus classes taught in a format emphasizing problem-solving in small groups. These students performed better on common final exam problems and passed the course at higher rates than their peers in lecture-based classes, though no significant difference was found between the two groups in terms of subsequent success in calculus. Because the treatment and comparison classes were taught by different professors, Collins suggested that future research should attempt to consider the importance of individual professors' instructional practices. Similarly, Gruber et al. (2021) reported that variability across different instructors' classes may have made some of their results difficult to interpret. These authors investigated a departmental redesign of some sections of an undergraduate precalculus course intended to increase active learning, defined as “classroom practices that lend themselves to student-to-student discussions of core mathematical concepts through engaging with complex tasks” (p. 359). The authors reported that failure and withdrawal rates across all sections of precalculus decreased following the partial redesign. However, the redesigned courses did not consistently have a lower percentage of failing grades compared to the courses that were not redesigned. The authors suggested that perhaps a lack of coordination across all precalculus sections may have contributed to this variability. This research implies that, while instructional practices that more actively engage students in problem-solving during class may be beneficial, it is important to consider how individual professors’ implementations of these practices might influence student outcomes.

Though less common than studies focused on academic success, some studies conducted in precalculus contexts have attempted to connect instructional practices with affective student outcomes or with students’ engagement in mathematical practices (e.g., Bowers et al., 2017; Cooper et al., 2017; Mkhatshwa, 2021; Zack et al., 2015). Zack et al. (2015) surveyed students who had learned precalculus in a format that regularly engaged students in collaborative problem-solving to investigate their perceptions of the class format and to track any changes reported in their attitudes over the course of the semester. Students generally expressed a dislike for the class format and, on average, reported negative shifts in attitudes at the end of the semester. The authors hypothesized these negative affective results were due, in part, to the students’ perceived lack of support from the professor; only 16.7% of students reported having had helpful interactions with their instructor during class. With a focus on mathematical practices, Bowers et al. (2017) investigated student perceptions of an active learning lab meeting that was added to a large lecture-based precalculus course in which students engaged in hands-on data collection and mathematical modeling activities grounded in course content. The authors surveyed students about their perceptions of their opportunities to engage in the eight Standards for Mathematical Practice (CCSSI, 2010) during these labs compared to the lecture-based class meetings. Students reported higher engagement in all eight practices during the labs than they did during lecture, particularly the first practice (“Make sense of problems and persevere in solving them”) and the third (“Construct viable arguments and critique the reasoning of others”). Students also reported positive feelings towards the active learning labs and indicated that they felt comfortable with the support provided by the lab instructors. These studies suggest instructor involvement may play a crucial role in influencing students’ attitudes and their perceptions of precalculus learning environments emphasizing active problem-solving.

Taken together, the studies reviewed here suggest that instructional practices can influence outcomes for precalculus students, and that the differential pedagogical choices individual professors make when implementing such practices are an important consideration. However, there is a lack of research exploring the potential connection between these pedagogical choices and student outcomes in precalculus, particularly students’ attitudes and perseverance. In the next section, we will review literature identifying pedagogical strategies that have the potential to positively influence these outcomes.

Pedagogies Influencing Students' Attitudes and Perseverance

Extant literature suggests that there are several pedagogical choices professors might make in their classrooms in order to nurture students' development of positive attitudes and perseverance. For example, consistently engaging and supporting students in problem-solving opportunities can support the development of perseverant actions. When instructors normalize student engagement with tasks warranting perseverance, rather than such challenging tasks being novel and rare, students can normalize learning from their mistakes and begin to value productive struggle (Bass & Ball, 2015; DiNapoli, 2019; DiNapoli & Miller, 2022; DiNapoli & Morales, Jr., 2021; Kapur, 2008; Jacobson et al., 2024; Morales, Jr. & DiNapoli, 2018). Further, when such problem-solving opportunities are collaborative, where students can work together and build off of each others' ideas, students often experience positive attitudinal shifts, such as increased enjoyment of mathematics (Noyes, 2012; Schettino, 2016) and increased motivation to persist in studying mathematics (Bressoud et al., 2015; Peterson, 2019). This suggests that professors should regularly engage students with challenging tasks in collaborative settings to help them develop strategies for persevering and to increase their mathematical enjoyment and motivation.

The use of scaffolding can also support the positive development of students' attitudes and perseverance. Anghileri (2006) posits several levels of scaffold to assist students in their problem-solving, all dependent on understanding students' thinking in the moment and/or how students have historically made sense of a mathematical idea. To help bridge a student's thinking from their current state of understanding to a future state, scaffolding can consist of restructuring a task into more manageable parts, modeling different problem-solving strategies in analogous tasks, asking targeted questions to the problem-solver based on their thinking, and/or encouraging students to self-scaffold by leveraging their prior knowledge to find ways to make progress toward a solution. Each of these types of scaffolds leverages formative assessment to understand student thinking and act on it. Furthermore, the latter two types of scaffolds can support students to initiate and sustain their productive struggle with the mathematics at hand (Reiser & Tabak, 2014). Investigating the influence of different scaffolding types on student perseverance, DiNapoli & Miller (2022) found that students persevered more, improved their self-confidence, and were more likely to overcome their perceived impasses when they recorded their own conceptualizations of problems and made connections to their prior knowledge at the outset of problem-solving. These perseverant outcomes did not occur as prevalently when students were not directly prompted to record their own conceptualizations, and students did not naturally transfer such self-scaffolding into problem-solving sessions without prompting. This suggests the importance of carefully designing learning environments to involve scaffolds that can encourage students' own conceptual thinking.

Finally, supporting students in making connections between mathematical concepts and real-world contexts that hold meaning for them has been connected to several positive attitudinal outcomes. Klee et al. (2022) argued that teaching mathematics through contextualized problems can make content more meaningful for students, increasing their sense of agency and thereby reducing math anxiety. Aikens et al (2021) found that biology majors who learned calculus in a course emphasizing how course concepts could be applied in biological contexts reported an increase in their perceived value of mathematics at the end of the semester. Similarly, Harris et al.'s (2015) engineering majors reported greater value and enjoyment of mathematics when their professors emphasized the connection between mathematical content and engineering applications. These studies highlight the importance of situating in-class activities in real-world contexts, particularly those aligning with students' interests or content from their majors.

Summary and Research Question

The literature reviewed suggests that regularly engaging students in collaborative problem-solving activities, surfacing and advancing student thinking using appropriate scaffolds, and situating in-class activities in meaningful real-world contexts can positively influence students' attitudes and perseverance. However, there is a lack of research exploring the extent to which these pedagogies are enacted by professors in precalculus contexts; inquiries must be made to study the precalculus instructors' role (e.g., Bowers et al., 2017; Gruber et al., 2021) in supporting students' attitudes and perseverance (e.g., DiNapoli & Miller, 2022; Klee et al., 2022). Therefore, we focused our study on the potential connection between individual precalculus professors' pedagogical choices and changes in their students' attitudes and perseverance, guided by the research question: *What are the relationships between professors' enactments of a research-based precalculus curriculum and changes observed in undergraduate students' attitudes towards mathematics and perseverance in problem-solving?*

Methods

This exploratory study employed a nonexperiemental research design (Newhart & Patten, 2023). We collected and analyzed several sources of empirical data to address our research question. Specifically, we conducted observations, surveys, interviews and problem-solving sessions to better understand our participants' actions in the setting of undergraduate precalculus.

Context and Participants

The precalculus curriculum in which this exploratory study was situated was *Precalculus: Pathways to Calculus* (Carlson et al., 2021). Our participants consisted of three precalculus professors (Prof-A, Prof-B, Prof-C) and 33 of their students. All professors in this study were teaching *Pathways* and, at the time of study, each had over two years of experience teaching *Pathways*.

This study took place at a public research university in the northeast United States. For context, this university enrolls approximately 23,000 students, is a Hispanic Serving Institution, and has an R2 Doctoral University status. Furthermore, this university ran approximately 40 sections of precalculus per academic year, which resulted in engaging approximately 1,200 students with *Pathways* annually. For additional context, approximately 77% of these students identified as non-white and 55% identified as women, both of which are groups traditionally underrepresented in STEM. Precalculus was the entry-level mathematics course for STEM majors at the university and was a coordinated course; each section used the same lesson plans, assignments, and exams.

Data Sources

To address all aspects of our research question, we collected data from four sources. First, we conducted two classroom observations for each participating professor. Each observation was video recorded and transcribed. Second, we conducted follow-up interviews with each professor to better understand their pedagogical choices apparent in the observations. Third, 24 student participants completed an Attitudes Towards Mathematics Inventory (ATMI; Tapia & Marsh, 2004) pre- and post-survey (at the start and conclusion of the semester, respectively), which measured students' mathematical self-confidence[1], value, enjoyment, and motivation. Fourth, 24 student participants engaged in 12 video-recorded problem-solving sessions each, engaging with one challenging task necessitating perseverance per session. The problem-solving sessions were facilitated by an interviewer, who simply reminded students to think-aloud as they were working. For transparency, 33 students participated in this study, but only 15 students participated in both the ATMI survey and the problem-solving sessions; 9 students participated in only the ATMI survey and 9 students participated in only the problem-solving sessions.

Data Analysis

To analyze the classroom observation data from each professor, we first divided the transcripts into 10- to 20-minute segments. We determined the parameters of these segments by shifts in instructional focus or classroom activity. Then, we used an observation rubric (adapted from Schoenfeld et al., 2014) to score each segment on three *Pathways*-aligned dimensions: Engaging and supporting students in problem-solving (PS), Understanding and advancing students' thinking (ST), and Providing opportunities for students to make connections (MC) (Figure 1). We used a 0-3 half-point scale, where 0 indicated no evidence of dimensional alignment and 3 indicated ample dimensional alignment. We averaged the segment scores for each dimension to get an overall score for each observation[2].

Figure 1

The Pathways Implementation Observation Rubric

***Pathways* Implementation Observation Rubric**		
Engaging and Supporting Students in Problem-Solving (PS)	Understanding and Advancing Students' Thinking (ST)	Providing Opportunities for Students to Make Connections (MC)
To what extent do classroom activities actively engage students in problem-solving? To what extent does the professor discuss, model, and encourage productive problem-solving practices?	*To what extent does the professor endeavor to understand students' thinking about content and respond in ways that help them to construct more productive understandings?*	*To what extent does the professor support students in making connections: to real-world contexts, across course content, and between functional representations?*

[1] The ATMI self-confidence dimension includes items addressing both self-efficacy and math anxiety, which were combined into the self-confidence dimension during factor analysis.

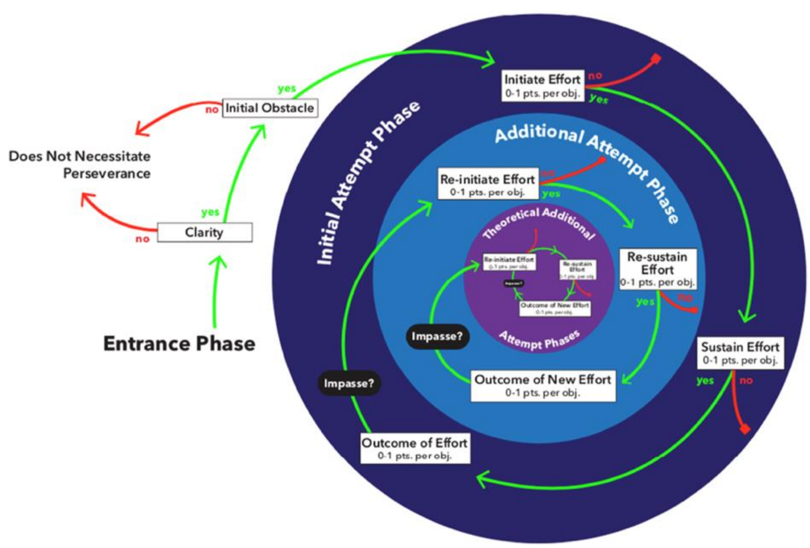
[2] Although an observation score of 3 in each dimension indicates a closer-to-ideal implementation of *Pathways*, it is important to note that maintaining a perfect level-3 alignment to all three dimensions at all times is likely impossible. For example, the time constraints of real-world classrooms make it difficult to always actively engage students in problem-solving.

To analyze the ATMI data, we averaged student responses to the pre- and post-survey along the four ATMI dimensions. Students responded to survey items on a 1-5 Likert scale, with 1 indicating low perceptions of mathematical self-confidence, value, enjoyment, and motivation, and with 5 indicating high perceptions. We calculated growth scores for each ATMI dimension, per student, then calculated aggregate scores for students in each professor's class.

To analyze the problem-solving session data, we used the Three-Phase Perseverance Framework (3PP; DiNapoli & Miller, 2022) to measure students' perseverance, or the extent to which students initiated and sustained, and re-initiated and re-sustained upon impasse, productive struggle on a challenging task (Figure 2). The 3PP operationalizes perseverance by considering the ways in which a student makes an initial attempt at solving a problem for which they do not already know a solution pathway, and makes an additional attempt at solving the problem if their initial attempt was unsuccessful and led to a perceived impasse (i.e., they were substantially stuck). Theoretically, a student could engage in multiple additional attempts as they encounter multiple impasses. In our analysis, we only considered students' experiences around one perceived impasse during the problem-solving. We used a points-based 3PP to capture if and how a student initiated (0-1 point) and sustained (0-1 point) efforts toward a solution before an impasse, and if and how those efforts were mathematically productive (0-1 point). After an impasse, we also captured if and how a student re-initiated (0-1 point) and re-sustained (0-1 point) their efforts toward a solution, and if and how those new efforts were mathematically productive (0-1 point). We relied on students' think-alouds as well as their written work to make scoring decisions.

Figure 2

The Three-Phase Perseverance Framework



Thus, students could earn 0-6 3PP points per problem-solving session, with 0 indicating no evidence of perseverance and 6 indicating ample evidence of perseverance. When considering students' growth, an improvement of just one

3PP point is substantial since it could represent perseverance growth in several ways, such as the difference between not engaging at all vs. initiating some effort (0 vs. 1) or engaging in a successful first attempt but giving up upon impasse vs. re-initiating a second attempt after impasse (3 vs. 4).

Each problem-solving participant engaged in 12 problem-solving sessions, each earning 12 3PP scores. We used linear regression to model each student's perseverance growth across their 12 problem-solving sessions, then found an aggregate model for each professor's students. Our results focus primarily on the slopes of these linear models; these 3PP slopes represent the average increase of each professors' students' 3PP scores per problem-solving session.

Results

We were able to observe marked differences in the pedagogical choices Prof-A, Prof-B, and Prof-C made while implementing the *Pathways* curriculum in their classrooms, as well as changes in their students' attitudes and perseverance over the semester. Prof-A's observation scores were generally lower across all three *Pathways* implementation dimensions, Prof-B's were generally higher, and Prof-C's scores were mixed (higher for their first observation than their second; Table 1). On average, Prof-A's students reported a slightly negative shift in attitudes, while Prof-B and Prof-C's students reported a slightly positive shift. Finally, all three professors' students improved in their perseverance over the semester, though Prof-B and Prof-C's students' growth in this category was about four to five times as great as that of Prof-A's students.

Table 1

Overall Results

		***Pathways* Implementation** (0-to-3-point scale)			**Average ATMI Changes** (Post-Pre, 5-point Likert Scale)					**3PP Slope** (Avg increase per session)
		PS	ST	MC	SC	V	E	M	Tot	
Prof-A	Obs 1	1.00	1.40	2.10	0.00	-0.31	-0.17	-0.25	-0.15	0.09
	Obs 2	1.00	1.14	1.70						
Prof-B	Obs 1	2.25	2.17	2.38	0.17	0.37	0.10	0.20	0.21	0.44
	Obs 2	2.00	2.31	2.40						
Prof-C	Obs 1	2.20	2.14	2.40	0.72	0.16	0.30	0.20	0.41	0.34
	Obs 2	1.70	1.97	1.40						

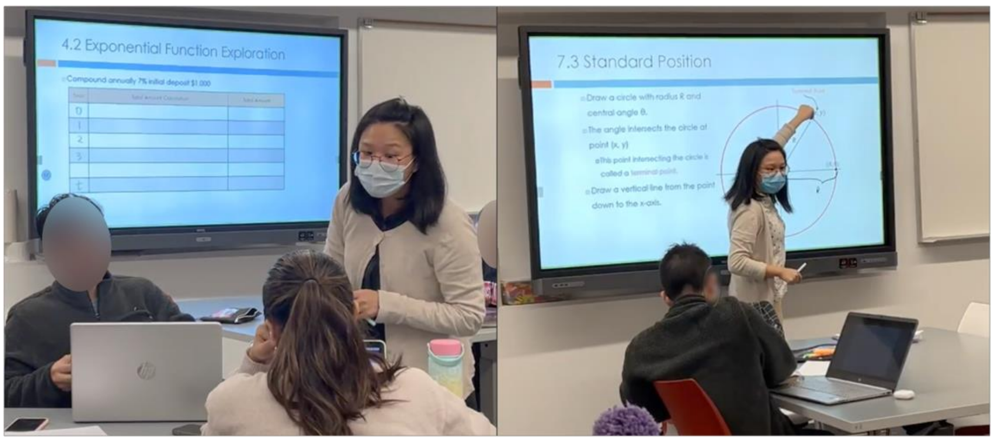
Note: PS: Engaging and supporting students in problem-solving; ST: Understanding and advancing student thinking; MC: Providing opportunities for students to make connections; SC: Self-confidence; V: Value; E: Enjoyment; M: Motivation.

Classroom Observation Results

In the PS category, Prof-C scored higher in their first observation than their second (an average of 2.20 vs 1.70). In their first observation, Prof-C dedicated over a third of the class meeting time to implementing and discussing one of *Pathways*' investigations, in which students worked in small groups deriving the formula for an exponential function by exploring an investment scenario. Throughout this activity, Prof-C's students were the primary problem-solvers in the room while Prof-C circulated among groups in a supportive role (Figure 3, left). In their follow-up interview, Prof-C indicated that this observation was one of the few times they were able to implement an investigation due to time constraints. In their second observation, Prof-C spent the majority of class time providing exposition at the board (Figure 3, right). Although the students were still engaged in problem-solving as they interacted with Prof-C, this engagement was less active than it had been during the first observation.

Figure 3

Prof-C's Two Observations



Prof-A's scores in the PS dimension were low for both observations because they rarely provided opportunities for students to actively engage in problem-solving. In their interview, Prof-A reported that they did not implement *Pathways* investigations or other collaborative group work because they were uncomfortable circulating among students due to COVID concerns. Prof-B's higher PS observation scores reflect their consistent use of interactive whole-class discussions to introduce new content and work example problems. Although we did not observe Prof-B implementing a *Pathways* investigation, they reported in their follow-up interview that they dedicated one class meeting per week for students to work collaboratively on *Pathways* investigations.

The differences observed among the professors in the ST category are well-illustrated by juxtaposing two moments from the second observations of Prof-A and Prof-B. On this day, Prof-A was discussing the signs of the *x*-coordinates of different terminal points along the unit circle:

Prof-A: As my angle gets bigger, if I measured this, what's this distance relative to the one down here, bigger or smaller?

Student-1a: Smaller.

Prof-A: Smaller. … And now as I move further, between pi over two and pi, this distance here is getting bigger because it's moving away. But because it's on the left-hand side, is it positive or negative?

Student-2a: Negative.

Prof-A: Negative. And so we're gonna get the same effect here.

This exchange was scored as ST level 1 because, while Prof-A did solicit input from their students, they asked questions that did not require students to explain their reasoning (i.e., to better understand students' thinking), and did not follow up on student answers (i.e., to help advance their thinking). In contrast, we observed the following interaction between Prof-B and their students regarding a problem about the decay rate of caffeine in the body:

Prof-B: So I know the 10-hour decay factor [is .2722], how can I use [this] to find the five-hour decay factor?

Student-1b: You could try and divide it by half. Because it's 10 hours and since finding the five hour one, you could just divide that by two.

Prof-B: That is a good point. … If we did, yeah, [Student-2b]?

Student-2b: Square root of .2722?

Prof-B: Ah. We should explore these options. … [Student-1b], would you mind telling me your reasoning one more time for this?

Student-1b: Well, since it's like a 10-hour decay, and they ask you for like the five-hour growth decay, I just thought if we just divided by two…

Prof-B: [Student-2b], what, what's your reasoning for this?

Student-2b: Because the five-hour times the five-hour would be the 10-hour.

Prof-B: [*Prof-B polls the class to see which answer they agree with; about half the class votes for each answer*.] … I want to talk about how we can use our intuition to maybe rule this out a little bit. So, if I keep about 27% of my caffeine intake over 10 hours, if I only look at five hours, will I have more than 27% or less than 27% in my body?

Student-3b: More.

Prof-B: In five, I would have more, because I need the full 10 hours to decay all the way down to 27%. So that is one logical issue that comes into play here, that if [.2722 divided by 2] was the five-hour growth or decay factor, this is … .1372, which is smaller than the 10-hour decay factor. So this means in five hours, I only have about 14% left in my system. But in 10 hours, I have more than that left in my system. That feels weird, right?

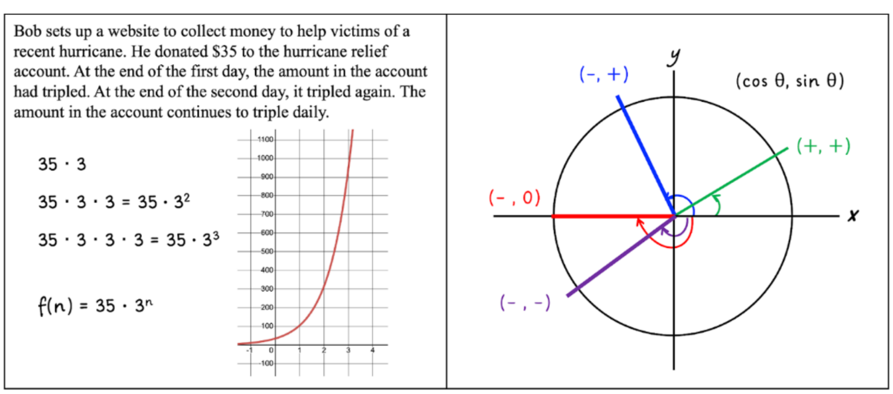
This exchange was scored as ST level 3, because Prof-B invited students to share their reasoning and polled the whole class to understand how students were thinking about the problem. Then, Prof-B introduced a method that could help students to advance their thinking by considering which answer was more reasonable in context.

We also observed Prof-B using scaffolded questioning to surface and advance student thinking in their first observation. Prof-B asked students to recall some of the attributes of symbolic representations of polynomial functions. When it became clear that students were having difficulty answering the question, Prof-B asked students if they could think of examples of polynomial functions, to which a student answered, "linear functions are polynomials, plus quadratics." Prof-B then asked students about attributes of the symbolic representations of linear and quadratic functions to help them build a list of polynomial attributes (i.e., whole number exponents, real coefficients). Later in the same observation, when students were unable to identify the leading monomial term of a polynomial, Prof-B helped the students break the phrase down (e.g., thinking about what the prefix *mono* indicates) until they were able to answer the original question. Prof-C likewise made frequent use of scaffolded questioning during their first observation while their students were working on the exponential function investigation. For example, Prof-C found that a student had used an investment formula to obtain a correct answer, but could not explain why the formula worked. Prof-C encouraged the student to connect what they knew about interest calculations to investigate the formula they had used, saying, "Do one year at a time. Do … year one, year two. And then when you get to year three, once you have at least three calculations written out, the pattern might start to emerge." In sum, both Prof-B and Prof-C frequently used scaffolded questioning during their observations, often to help students access and leverage their prior mathematical knowledge.

In the MC observation dimension, Prof-A scored higher on their first observation than on their second (2.10 and 1.70, respectively). During the first observation, they developed the $f(x) = a \cdot b^x$ form of an exponential function through the investigation of a real-world problem context (Figure 4, left). Throughout their exposition, Prof-A also mentioned connections between the symbolic and graphical representations of the function, and contrasted both representations with those of linear functions. Although Prof-A presented the day's material in a highly-connected manner, their score averaged closer to 2 than 3 because they explained these connections rather than inviting students to explore them for themselves. In their second observation, instead of developing content via a contextual exploration, Prof-A first presented a lesson on the signs of the coordinates of the terminal points around the unit circle (Figure 4, right) before applying these concepts in word problems. Prof-A also did not explore connections to other course concepts or across functional representations to the same extent as they had in their first observation. Some of the difference between Prof-A's first and second MC scores is likely attributable to the nature of the content covered on each day. However, these scores provide a window into the relative emphasis placed on connection-making during both observations.

Figure 4

Prof-A's Presentations of an Exponential Function and Unit Circle Coordinates



We observed an interesting MC moment in Prof-B's second observation when working with examples of exponential growth. Prof-B said, "I need some inspiration for an example. … What's an experience that someone's had [this weekend]?" When a student suggested Thanksgiving foods as a context, Prof-B spontaneously created a problem about the price of cranberries increasing exponentially during the holiday season. When asked about this moment during their follow-up interview, Prof-B indicated that this was a common practice of theirs:

> I want to bring this into a context that [students] are actively concerned with, maybe it's something you did right before class … maybe it's one of your really strong interests. … Students seem really excited about sharing things about themselves … so it's injecting the relevance of mathematics into their lives without beating them over the head with it.

Prof-C also provided several opportunities for students to make connections between content and their knowledge outside of class, particularly in their first observation. For example, Prof-C had several biology and chemistry majors in their class. When the class was discussing whether or not the base of an exponential function could be a negative number, Prof-C asked students to discuss whether a negative base would make sense in the context of bacteria growth or radioactive decay.

Attitudes Towards Mathematics Results

We generally saw small shifts in students' attitudes over the course of the semester, though some differences between the three professors' students are notable (Table 1). Generally speaking, Prof-A's students reported small but negative shifts in attitudes in all ATMI categories except for self-confidence, in which they reported no change. In other words, students in Prof-A's class reported a decrease, on average, in how much they valued mathematics, how much they enjoyed mathematics, and in their motivation to learn mathematics. Prof-B and Prof-C's students reported positive shifts in all four ATMI categories. Students in Prof-B and Prof-C's classes reported an increase, on average, in their self-confidence with mathematics, in how much they valued mathematics, how much they enjoyed mathematics, and in their motivation to learn mathematics. Prof-B's students' most pronounced shift was in their reported value of mathematics, which increased by an average of 0.37 on the 5-point scale. Prof-C's students' increase in items regarding their mathematical self-confidence (0.72 on the 5-point scale) was the largest shift in any ATMI dimension for the three professors' students. Collectively, these findings suggest that students in Prof-B and Prof-C's classes may have experienced a positive attitude shift towards mathematics over the course of the semester. This is juxtaposed with students in Prof-A's class, who may have experienced a negative attitude shift toward mathematics over the course of the semester.

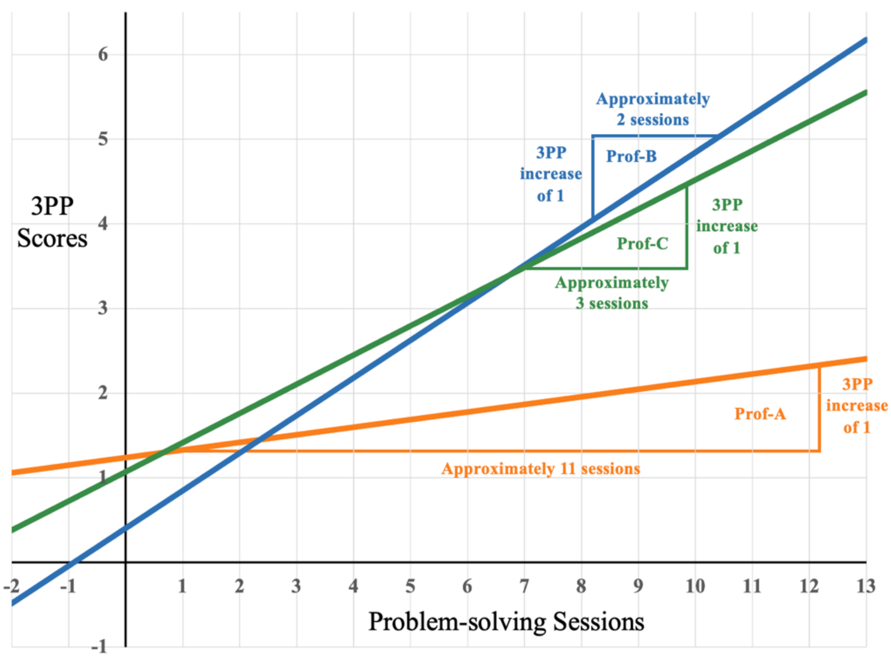
Perseverance in Problem-Solving Results

On average, all students in the three professors' classes showed perseverance growth, as evidenced by positive 3PP slope scores (Table 1). Students in Prof-A's class increased in their perseverance by 0.09 3PP points per problem-solving session. Students in Prof-B's class increased in their perseverance by 0.44 3PP points per problem-solving session, approximately 4.9 times greater growth compared to Prof-A's students. Students in Prof-C's class increased in their perseverance by 0.34 3PP points per problem-solving session, approximately 3.8 times greater growth compared to Prof-A's students.

One way to interpret these 3PP slopes is to consider the number of problem-solving sessions it would take, on average, for a student in a class to improve their perseverance by one 3PP point. Figure 5 shows a graph of the linear models of students in each professor's class, relative to their perseverance growth, with a focus on their slopes and how to interpret them. For students in Prof-A's class, the slope of 0.09 suggests it would take approximately 11 problem-solving sessions for a student to show one point of 3PP growth. For students in Prof-B's class, the slope of 0.44 suggests approximately 2 problem-solving sessions for a student to show one point of 3PP growth. For students in Prof-C's class, the slope of 0.34 suggests approximately 3 problem-solving sessions for a student to show one point of 3PP growth.

Figure 5

Graphs and Interpretations of Average Perseverance Growth, by Class



	3PP Slope (Avg increase per session)
Prof-A	0.09
Prof-B	0.44
Prof-C	0.34

Illustrative Example of Perseverance Growth

To illustrate how one student's perseverance increased across two problem-solving sessions, consider Student-4b's experiences in their second and seventh problem-solving sessions. Student-4b was a student in Prof-B's precalculus class and they earned 3 3PP points and 6 3PP points in their second and seventh problem-solving sessions, respectively.

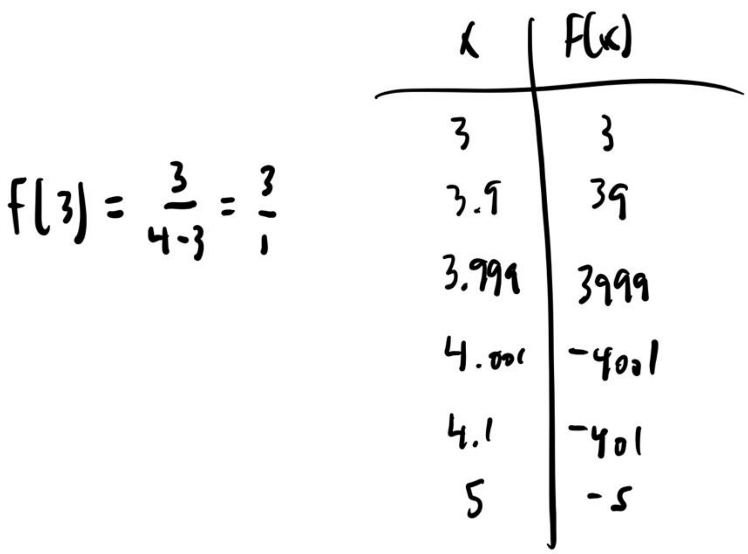
During their second problem-solving session, Student-4b was working on the following task involving limits:

$$\text{Find } \lim_{x \to 4} f(x) \text{ when } f(x) = \frac{x}{4-x}$$

Limits were a relatively new concept for Student-4b, and, during their think-aloud problem-solving session, they confirmed that this task was indeed challenging and that they did not immediately know a solution pathway. In their Initial Attempt Phase of the 3PP, Student-4b initiated their effort (1 3PP point) with this task by expressing that they could "plug some numbers into [the function] to see what happens." Student-4b sustained this effort (1 3PP point) by first finding $f(3) = 3$, and continuing such efforts by constructing a table of values and implicitly testing values around but not including $x = 4$ (Figure 6). This implied a mathematically productive outcome of that sustained effort (1 3PP point) because Student-4b's table of values suggested that they suspected something interesting about the behavior of $f(x)$ at $x = 4$.

Figure 6

Student-4b's Sustained Efforts in their Initial Attempt during their Second Problem-Solving Session



Next, Student-4b reached a perceived impasse. After studying their table of values for a minute, they admitted to the interviewer, "Ok. I don't get it now. So what?" The interviewer prompted Student-4b by asking, "Ok, what are you thinking now?" Student-4b stated, "I don't know, I guess I'm done." This concluded Student-4b's efforts, and they earned 3 3PP points, reflecting a quality first attempt at solving the task, but no additional attempt after impasse.

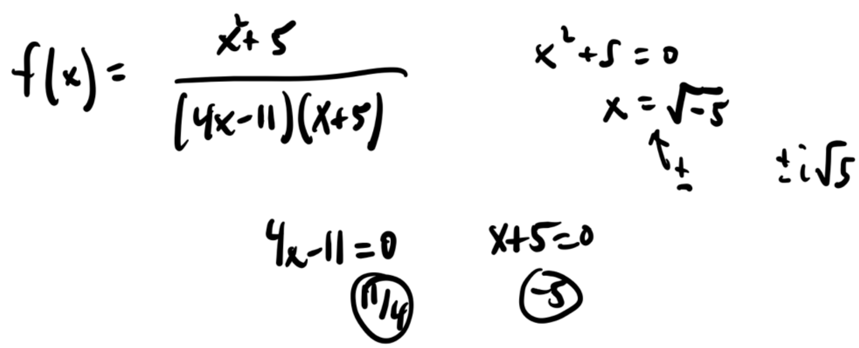
Weeks later, during their seventh problem-solving session, Student-4b was working on the following task involving roots and asymptotes:

$$\text{Find all roots and asymptotes of } f(x) = \frac{x^2+5}{(4x-11)(x+5)}$$

Similar to the limit task in their second problem-solving session, roots and asymptotes were relatively new concepts for Student-4b and this was the first time they were presented with a task prompting them to find roots and asymptotes all in one task. Student-4b confirmed this task was a challenge for them, saying they were "overwhelmed" because it was "asking so much," indicating that they did not immediately know a solution pathway for this task. In their Initial Attempt Phase of the 3PP, Student-4b initiated their effort (1 3PP point) by setting the expression in the numerator of $f(x)$ equal to zero while saying "I could do this." Student-4b sustained their effort (1 3PP point) by solving that equation and setting the factored expression in the denominator equal to zero and solving (Figure 7). While Student-4b was solving, they revealed while thinking aloud, "I don't really know why I should be doing this." Despite their confusion about what the solutions to those equations meant, these sustained efforts implied a productive mathematical outcome (1 3PP point) since the solutions were related to the roots and vertical asymptotes of $f(x)$.

Figure 7

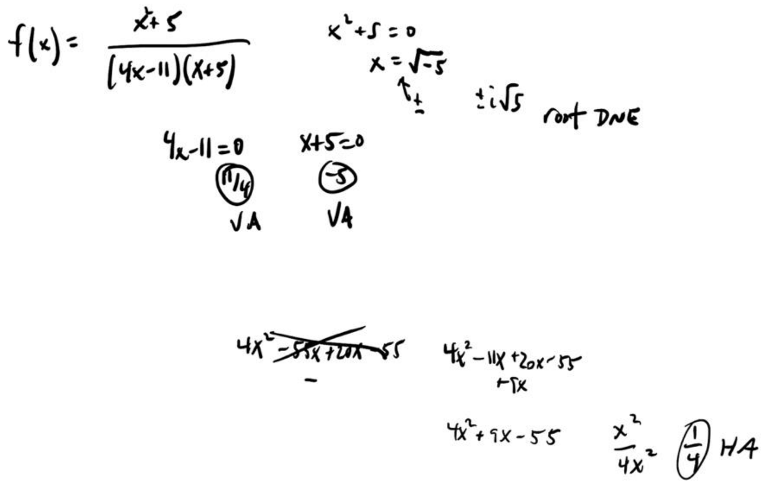
Student-4b's Sustained Efforts in their Initial Attempt during their Seventh Problem-Solving Session



Next, it became clear Student-4b had reached a perceived impasse. Student-4b said, "I don't really know what to do next." After a few seconds, the interviewer asked "Ok, what are you thinking about now?" Student-4b paused for a few seconds, looking over their work. Then, they exclaimed, "Wait! Can I look in my [note] book?" Starting their Additional Attempt Phase of the 3PP, Student-4b then re-initiated their effort (1 3PP point) toward a complete solution by retrieving their precalculus notebook. After paging through it for a few seconds, they shared, "Ok, so when [the numerator] is zero that means it's all zero. And [the denominators] are vertical asymptotes, you know, dividing by zero." This suggested that Student-4b leveraged an existing resource, their handwritten notes from class, to help them overcome their perceived impasse and make an additional attempt. They re-sustained their effort (1 3PP point) by labeling their earlier solutions to clarify that $f(x)$ had no roots and two vertical asymptotes (Figure 8). Additionally, Student-4b further sustained their effort by beginning some procedures toward finding a horizontal asymptote: they began to multiply the binomial expressions in the denominator of $f(x)$. When prompted to think aloud, Student-4b said, "I have to get [the denominator] as an x-squared" and went on to rewrite the expression in the denominator in standard form. Student-4b finished their efforts by simplifying the ratio of the leading terms in the numerator and denominator, respectively, and clarified where $f(x)$ has a horizontal asymptote (Figure 8). These re-sustained efforts clearly illustrated a new productive mathematical outcome (1 3PP point) since Student-4b was able to correctly solve this problem. This concluded Student-4b's efforts in their seventh problem-solving session, earning 6 3PP points, reflecting a quality first attempt and additional attempt at solving the task, including overcoming a perceived impasse.

Figure 8

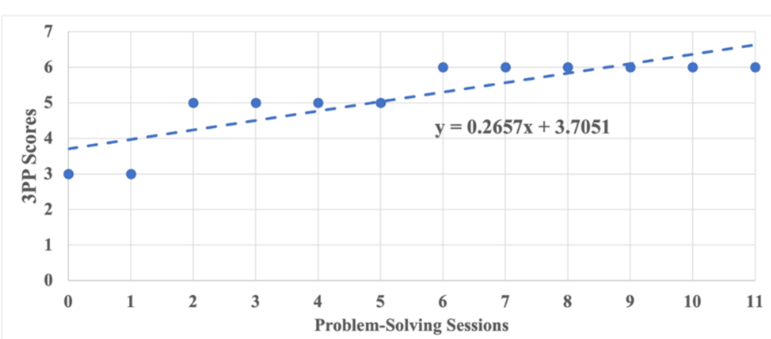
Student-4b's Re-Sustained Efforts in their Additional Attempt during their Seventh Problem-Solving Session



This illustrative example of Student-4b's experiences across two problem-solving solving sessions shows some possible ways in which students can improve their perseverance over time. Student-4b's engagement in their second problem-solving session helped show how students could be presented with a challenging task they do not know how to solve and still engage and make mathematical progress, despite eventually encountering a debilitating perceived impasse. Student-4b's engagement in their seventh problem-solving session suggested meaningful growth in perseverance, because they were able to leverage a personal resource to help them overcome a perceived impasse and re-engage productively with a challenging task. For reference, Figure 9 shows Student-4b's 3PP scores across all their problem-solving sessions, as well as their linear model.

Figure 9

Student-4b's 3PP Scores per Problem-Solving Session



Discussion

Although this study was exploratory, our analysis helped reveal some potential connections between different ways of implementing the *Pathways* curriculum and undergraduate students' changes in attitudes and perseverance. In short, professors' pedagogical choices might help explain attitude development and perseverance growth in their students. Moreover, our results and interpretations of results were largely consistent with what is implied from the literature.

First, we saw evidence of a potential connection between professors implementing *Pathways* with a focus on problem-solving and students' attitudes and perseverance growth. Our results suggest that when the professors consistently engaged their students in problem-solving or indicated this as a normalized practice in their classroom, their students were better supported in positive attitude shifts and improvements in their perseverance. We note Prof-A's lower PS observation scores compared to Prof-B and Prof-C, and the corresponding lower ATMI scores and 3PP slope scores. Prof-B and Prof-C's dedication, or partial dedication, to consistently implementing the *Pathways* investigations may help explain why we saw more evidence of problem-solving, on average, in their classrooms compared to Prof-A. Prof-A's COVID concerns may partially explain these results because Prof-A reported feeling uncomfortable circulating around the classroom with students working in groups, on investigations or otherwise. Through this lens, Prof-A's lack of focus on problem-solving during their implementation of *Pathways* was more of a safety measure than a pedagogical choice. However, Prof-B and Prof-C's students still spent much more time actively problem-solving during class and, consistent with the literature, we saw those students experience positive shifts in attitudes and faster improvement in perseverance (e.g., Barnes, 2019; DiNapoli, 2019; DiNapoli & Miller, 2022; DiNapoli & Morales, Jr., 2021; Morales, Jr. & DiNapoli, 2018; Noyes, 2012; Peterson, 2019; Schettino, 2016; Schoenfeld, 1992).

Student-4b's illustrative example of perseverance growth helps exemplify this potential connection between a classroom focus on problem-solving and students' attitudes and perseverance growth. Student-4b was a student in Prof-B's class, and they leveraged their own class notes to help overcome an impasse during problem-solving on a challenging task during their seventh problem-solving session. Student-4b's usage of their own resource, their personal notebook, during a time of uncertainty during problem-solving helps support the idea that they valued their perception of these mathematical ideas and had the self-confidence to believe they could overcome an obstacle using a self-created mathematical resource. Notably, Student-4b did not leverage a similar resource upon impasse during their second problem-solving session. Aligned with the findings that associated improved perseverance with normalized classroom opportunities for productive struggle (e.g., Bass & Ball, 2015; DiNapoli, 2019; DiNapoli & Miller, 2022; DiNapoli & Morales, Jr., 2021; Kapur, 2008; Jacobson et al., 2024; Morales, Jr. & DiNapoli, 2018), Student-4b's example suggests a connection between developing attitudes and perseverance growth, perhaps influenced by the more consistent exposure to problem-solving in Prof-B's class.

Second, we saw evidence of a potential connection between professors implementing *Pathways* with a focus on understanding and advancing student thinking and students' attitudes and perseverance growth. Our results suggest that when professors showed evidence and intent to use scaffolded questioning to understand how their students were thinking about the mathematics during instruction, their students displayed more productive attitude shifts and steeper growth in their perseverance. Compared to Prof-B and Prof-C, we note Prof-A's lower ST observation scores and the related, lower student ATMI and 3PP slope scores. Moments from Prof-A's second observation, between Prof-A and Student-1a and -2a, help showcase teaching moves that may not elicit/advance student thinking. Conversely, moments from Prof-B's second observation, between Prof-B and Student-1b, -2b, and -3b, help showcase scaffolded questioning techniques that personally invited student thinking into the conversation and helped advance student thinking about exponential decay. We do not contend this to be a highly-correlated, one-to-one relationship. However, we do note that these results are consistent with the literature on these kinds of connections, namely that personal attention from an instructor during class can nurture students' positive attitudes (Zack et al., 2015) and a pedagogical focus on using scaffolding to understand and advance student thinking can help improve students' self-confidence, motivation, and perseverance (DiNapoli, 2019, 2023; DiNapoli & Miller, 2022; Reiser & Tabak, 2014).

Again, Student-4b's illustrative example of perseverance growth helps demonstrate this potential connection between a classroom focus on understanding and advancing student thinking and students' attitudes and perseverance growth. As Prof-B's student, Student-4b was exposed to pedagogical practices involving scaffolded questioning and helping students make prior knowledge connections to better understand a mathematical situation. We see a connection between such pedagogy by Prof-B and the self-scaffolding observed in Student-4b's seventh problem-solving session when Student-4b leveraged their personal notebook to help overcome an impasse during problem-solving on a challenging task. The benefits of self-scaffolding have been documented in the perseverance literature (DiNapoli, 2019, 2023; DiNapoli & Miller, 2022), yet those participants did not naturally transfer those self-scaffolds into problem-solving situations where they were not explicitly asked to do so. In this study, there was evidence of Student-4b transferring self-scaffolding techniques into their problem-solving sessions without prompting. One possible explanation is that Student-4b was being consistently engaged in scaffolded questioning during Prof-B's class, and Prof-B may have been modeling a specific problem-solving strategy for Student-4b to emulate on their own, that is, to explicitly attend to one's [notebook of] prior knowledge when at an impasse.

Lastly, we saw some evidence related to Prof-B regarding a potential connection between implementing *Pathways* with a focus on students making connections and their attitudes growth. Our results suggest that Prof-B's prompting of students' ideas for word problem contexts may be related to students in Prof-B's class showing the greatest improvement in the value of mathematics dimension of the ATMI. We saw some similar opportunities in Prof-A and Prof-C's classes, but none so personal to students. These results may extend Aikens et al. (2021), in that providing students opportunities to make connections between content and real-world contexts may increase students' perception of the value of mathematics, especially when those connections are personal.

Conclusion and Limitations

This exploratory study examined the influence of professors' implementation of a research-based precalculus curriculum, *Pathways*, on students' attitudes and perseverance. Results showed varying pedagogical approaches among professors, with students of Prof-B and Prof-C demonstrating positive attitude shifts and greater perseverance growth compared to those of Prof-A. These results underscore the influence of professors' instructional strategies on the potential benefits of the *Pathways* curriculum in fostering positive attitudes and improving perseverance among undergraduate students. However, our study was limited by its exploratory nature. Based on our limited data, we could not make substantial claims about professors' pedagogical practices and their influences on student outcomes. Instead, we only posited potential relationships between how professors may implement *Pathways* in their classrooms and the related attitude development and perseverance growth in their students. Future research will examine more *Pathways* instructors, their classrooms, and their students, and refine our data collection and analysis plans. By observing more precalculus instructors more often, and by collecting more student data from their classrooms, we will be able to substantiate the relationships between professors' pedagogies and students' attitudes and perseverance. In all, such future research will help us better understand the nature of our precalculus courses, and help us find ways to better support our precalculus students in this STEM-gateway course.

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Ethics Statement: We confirm that this research complies with the ethical parameters adopted by the Institutional Review Board at Montclair State University (Study #: IRB-FY21-22-2484; Approval Date: January 19, 2022). Furthermore, all research procedures were consistent with the principles of the research ethics published by the American Psychological Association.

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