JOURNAL OF RESEARCH IN SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION
DOI: 10.31756/jrsmte.514

How could the Presentation of a Geometrical Task Influence Student Creativity?

Athanasios Gagatsis & Iliada Elia

University of Cyprus, Cyprus

Zoi Geitona

National and Kapodistrian University of Athens, Greece

Eleni Deliyianni

Cyprus Ministry of Education, Culture, Sport and Youth, Cyprus

Panagiotis Gridos

University of the Aegean, Greece

Abstract: This study aims to investigate high school students' geometry learning by focusing on mathematical creativity and its relationship with visualisation and geometrical figure apprehension. The presentation of a geometrical task and its influence on students' mathematical creativity is the main topic investigated. The authors combine theory and research in mathematical creativity, considering Roza Leikin's research work on Multiple-Solution Tasks with theory and research in visualisation and geometrical figure apprehension, mainly considering Raymond Duval's work. The relations between creativity, visualization and geometrical figure apprehension are examined through four Geometry Multiple-Solution Tasks given to high school students in Greece. The geometrical tasks are divided into two categories depending on whether their wording is accompanied by the relevant figure or not. The results of the study indicate a multidimensional character of relations among creativity, visualization and geometrical figure apprehension. Didactical implications and future research opportunities are discussed.

Keywords: *Creativity; Geometrical figure; Geometry; Multiple-solution tasks; Visualisation.*

Introduction

Creativity has been recognized as one of the most important skills of the 21st century. Although its importance has been widely recognized, it is also agreed that creativity is a complex property of the human mind. Mann (2006) has underlined its complex nature by stating that, in the existing literature, there are over 100 definitions given to the term. Guilford (1967) has defined creativity as the ability to solve problems in multiple ways. Krutetskii (1976) and Singh (1987) have given similar definitions to mathematical creativity by relating the term to independence and originality and innovation and novelty, respectively. In the field of Mathematics, creativity has been underlined as a skill which should be taught to students through school curricula (NCTM, 2000). Another important addition to the existing literature came from Kaufman and Beghetto (2009) who stated that creative ability is not a "charisma" of a small group of talented or gifted students but instead, it is an ability which can be fostered to all students through targeted teaching and learning. Sawyer (2015) has further highlighted the need for students to go beyond facts and procedural knowledge in order for them to cultivate their creative ability. As proved by numerous studies (e.g. Ervynck, 1991; Silver, 1997; Stupel & Ben-Chaim, 2017), linking mathematical ideas using multiple approaches for solving problems or proving statements strengthens mathematical reasoning, improves comprehension and therefore increases student creativity in Mathematics.

In Mathematics, Multiple-Solution Tasks (MSTs) have proved to be a useful teaching and learning tool which can not only foster student creativity but also, measure it (e.g. Leikin, 2014; Levav-Waynberg & Leikin, 2012a; 2012b). As Levav-Waynberg & Leikin (2012a) have stated, MSTs in geometry give students the opportunity to investigate numerous solutions by applying knowledge and concepts already taught in school geometry curricula.

Most geometrical thinking is based on spatial visualization and reasoning (Dindyal, 2015; Van den Heuvel-Panhuizen & Buys, 2008) and this is the reason why we propose that both the concepts of visualisation and geometrical figure apprehension are closely related to creativity in Geometry. As figures are a major part of Geometry, visualization is an important part of geometrical thinking. Visualization is related to the ability to recognize and identify figural units in a configuration of shapes and also, the ability to perform operations with figures (Duval, 1995). The figure belongs to a special semiotic system, which is connected to the perceptual visual system, obeying to internal rules of organization. In Geometry, three semiotic registers are used; the natural language, the symbolic language and the graphical. These three registers lead to three different cognitive activities on behalf of students; (a) seeing and recognizing shapes (b) measuring magnitudes and comparing and (c) understanding the figure through the given symbols and words (Duval, 2014). Furthermore, as figures are visual representations, figure apprehension is another factor taken into consideration in this study (Gagatsis, 2015). The types of geometrical figure apprehension are analysed in the following sections and their relation to creativity is investigated. The present article aims to shed new light on the relationships among high school students' mathematical creativity, visualization and geometrical figure apprehension.

Theoretical Framework

Visualization and Geometrical Figure Apprehension

Mathematical objects and more specifically, geometrical objects are abstract. For this reason, it is only through their semiotic representations that they can be studied. According to Gutiérrez (1996) mental images are "any kind of cognitive representation of a mathematical concept or property by means of visual or spatial elements" (Gutiérrez, 1996, p. 9) and they play a crucial part in visualization. Although the term "visualization" has various definitions and uses in the existing relevant literature, in this article, we decided to embrace a broader approach to the definition of the term suggested by Arcavi (2003), which is the following;

"Visualization is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas, and advancing understandings" (Arcavi, 2003, p. 217).

Duval (2014) has pointed out a potential difficulty relating to visualization in geometry. This difficulty has to do with the inter-relation between shape and geometrical properties and object recognition. For example, the perceptual recognition of the shape of a geometrical figure might mislead the recognition of its properties and of the geometrical object in general. Visualization however, overcomes the perceptual understanding of a figure (Duval, 2014). In other words, it allows students to visually distinguish among all the different figural units within a figure. Visualization therefore can lead to deeper figure apprehension.

A concept that is strongly related to visualization is spatial ability. According to Halpern (2000) the subjects of Mathematics such as Geometry, Topology and Trigonometry but also the nature of the mathematical subject as a whole, requires spatial ability. Elliot and Smith (1983) have noted that the various different definitions that have been given to spatial ability have led to different meanings of the term (Elliot & Smith, 1983). However, it is generally accepted that spatial ability involves the retrieval, retention and transformation of visual information in a spatial context (Halpern, 2000). According to Carroll (1993), spatial ability consists of five dimensions:

- Spatial Visualization is an individual’s ability to mentally rotate objects or their parts in the 3D space.
- Spatial Relations have to do with an individual’s ability to perceive an object from different positions and is often assessed through tasks of rotation or reflection in speed tests (Lohman, 1988).
- Closure Speed factor has to do with the ability to access spatial representations in long-term memory when incomplete or obscured cues are presented (in this case, subjects are not told what to look for).
- Closure flexibility is reflected in an individual’s ability to identify hidden patterns or figures in complex patterns.
- Perceptual Speed factor refers to the speed in identifying a given configuration among various given materials. For example, the subject may be asked to locate a unique item among a group of identical items, compare pairs of items or find a visual pattern.

Spatial thinking is related to geometrical figure apprehension (Panaoura, Gagatsis, Lemonides, 2007; Gridos, Gagatsis, Deliyianni, Elia, & Samartzis, 2018; Gridos, Avgerinos, Deliyianni, Elia, Gagatsis, & Geitona, 2021). Moreover, Gagatsis and Kalogirou (2013) argue that spatial ability is an important predictor in the way in which students apprehend the geometrical figure. Duval (1995) distinguishes among four types of apprehension; (a) perceptual apprehension refers to the recognition of a shape at first glance and the ability to name figures and to recognize several sub-figures in the perceived figure, (b) sequential apprehension is required whenever one must either construct a figure or describe its construction, (c) discursive apprehension is related to the fact that mathematical properties represented in a drawing cannot be determined through perceptual apprehension, but the geometrical properties must remain under the control of statements like definitions, theorems, etc and (d) operative apprehension is necessary in order to get an insight to a problem solution when looking at a figure. Operative apprehension depends on the various ways of modifying a given figure and one of these modifications may lead to the solution of a problem. These modifications can either be performed mentally or physically. Duval’s (2014) types of apprehension can be categorized in two approaches; the perceptual approach which, as explained above, is related to the instant recognition of the figure and the mathematical approach which relates to the operative apprehension of the geometrical figure.

Based on Gutiérrez’s (1996) and Yakimanskaya’s (1991) studies on visualization, mental images are linked to the external representations of figures. That is, mental images are created based on an individual’s interpretation of external representations or objects which are expressed in either verbal or graphical form. Thus, in the scenario in which a Geometry task is expressed verbally and the correspondent figure is not given, two levels of visualization could be distinguished. The first level involves the conversion of the verbal expression into the construction of the correspondent figure. The second level involves the transition from the geometrical figure to the solution of the problem. In other words, in such a scenario, both the operative and discursive apprehensions of a figure are involved.

Mathematical Creativity and Multiple Solution Problem Solving

Creativity is a complex mental ability, for which several definitions have been given in the research bibliography (Haylock, 1987; Leikin, 2009). The definitions differ according to whether they focus on the properties of the creative act and product (e.g Silver, 1997) or on stages of the creative processes (Ervynck, 1991). A very useful definition for the purpose of this study has been suggested by Torrance (1994). Torrance defines creativity as a multidimensional concept. According to him, fluency, flexibility, novelty (or originality) and elaboration, are different aspects of creativity. Fluency is related to raising multiple ideas (solutions) and the ability of a student to switch among different solutions. Flexibility is related to the number of new solutions when one solution has been found. As the word itself suggests, novelty refers to whether the solutions produced are original and unique. Finally, elaboration is related to one's ability to describe and generalize ideas. Silver (1997) has further contributed to Torrance's definition by suggesting ways to strengthen each creativity component. Fluency can be promoted by investigating multiple solutions for one problem. Flexibility is strengthened when an individual finds one solution but is encouraged to find more. Finally, originality is cultivated when an individual finds a new solution, which is different to solutions that were already known to him/her.

Numerous reseachers have investigated the relationship between mathematical creativity and mathematical problem solving. Silver (1997) and Chiu (2009) have both identified a connection between the two while other researchers, such as Mackworth (1965) have argued that there is a strong association between creativity and problem solving (and problem posing). To further highlight the relation of mathematical creativity with problem solving, Stupel & Ben-Chaim (2017) stated that a person's individual creativity increases by using multiple approaches for the solution of problem. This happens because, in the process of finding multiple solutions, both deeper understanding and mathematical reasoning are strengthened. Hiebert & Carpenter (1992) have described mathematical understanding as a network of connected concepts, properties and representations. It is these connections of mathematical ideas that can lead to successful (multiple) problem solving in mathematics, in contrast with rote memorization of facts.

Leikin, Levav-Waynberg, Gurevich, and Mednikov (2006) and Leikin and Levav-Waynberg (2007) defined Multiple Solution Tasks (MST) as tasks that explicitly request students to find more than one solution path to a specific mathematical problem. Various paths to solutions can be achieved in the following ways; (a) different representations of a mathematical concept; (b) different properties (definitions or theorems) of mathematical concepts from a mathematical topic; or (c) different mathematics tools and theorems from different branches of Mathematics. It is quite common among researchers (Bicer, 2021) to use different representations as a way to study students' mathematical creativity in Geometry. In this study, we are going to use both (a) different representations and (b) different properties (i.e. different geometrical figures which refer to different properties). The reason for choosing this tool is that different representations of geometrical figures, patterns and concepts specifically allow for explicit ways to show creativity in geometrical problem solving (Bicer 2021). It is important to note here that in the case of MSTs in Geometry specifically, different auxiliary constructions are considered as a difference of type (b) (Leikin, 2009). Leikin and Elgrabli (2015) identify two separate factors that determine the level of complexity of the auxiliary construction. The first has to do with the location of the auxiliary construction and more specifically, if it is within or outside the given figure and the second is related to the number of constructions which are necessary for a specific property to be recognized. In order to investigate students' performance in geometrical problem solving Leikin (2007) identified and defined the notion of solution spaces. According to Leikin (2007), each MST has its expert solution space which is the total number of possible solutions to a problem that is known at a particular time. In this study, we identify the collective solution space for each Task given to students which is the number of solutions found by the group of students who participated in this research.Leikin (2009) further advanced her research by proposing a scoring scheme for measuring each creativity component as well as total creativity based on student performance in MSTs. This scoring scheme will be used in our study and is further analyzed in the results section.

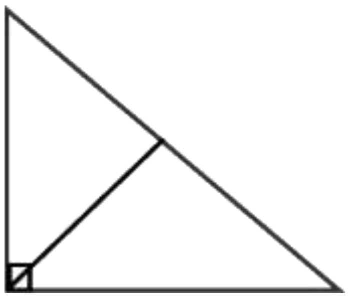
Moving a step forward, the study aims to shed new light as far as MSTs are concerned on evaluating and developing creativity in geometry considering the role of geometrical figure apprehension (Duval, 1995) and visualization. The usefulness of the geometrical figure in problem solving in geometry is beyond doubt as it provides an intuitive presentation of all constituent relations of a geometrical situation (Duval, 1995).

The Role of Representations in Understanding the Wording of a Mathematical Task

Tables, graphs, equations and other tools which are used for the depiction of mathematical ideas are called "representations" (Confrey & Smith, 1991). Numerous researchers suggest that students can understand mathematical concepts if they experience them through various mathematical representations. (e.g. Sierpinska, 1992; Gagatsis & Shiakalli, 2004; Duval, 2006; Gagatsis, Christodoulou, & Elia, 2013; Gagatsis, Deliyianni, Elia, Panaoura, & Michael-Chrysanthou, 2016; Nicolaou, Gagatsis, Panaoura, Deliyianni, Elia, & Televantou, 2020). It is also reported that the role of representations is very important in problem solving (Elia, Gagatsis, & Demetriou, 2007). It is worth noting here that there is a negative phenomenon related to the use of representations in mathematical tasks which is called "compartmentalization". The problem that occurs is that groups of tasks are classified by the students during solving different tasks according to the representations of the tasks and not according to the mathematical concepts involved (Elia, Gagatsis & Gras, 2005; Elia, Panaoura, Eracleous, & Gagatsis, (2007).

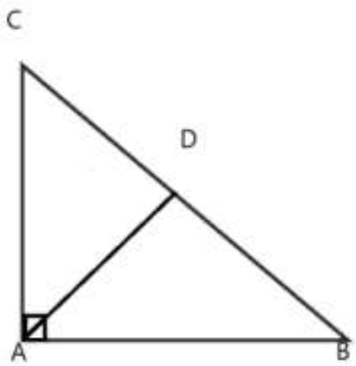
Taking into consideration the fact that three different semiotic registers intervene in geometry, that is the natural language, the symbolic language and the graphical register, there are different forms of presentation of a geometrical task based on the different combinations of elements from the three registers. Thus, for example Task 1 in our research could be presented in the following alternative ways:

1. Prove in as many ways as you can that the median drawn to the hypotenuse of the right triangle equals half the hypotenuse.



2. Prove in as many ways as you can that the median drawn to the hypotenuse of the right triangle equals half the hypotenuse.

3. Prove in as many ways as you can that the median AD of a right triangle ABC (CD=DB), equals half the hypotenuse CB.



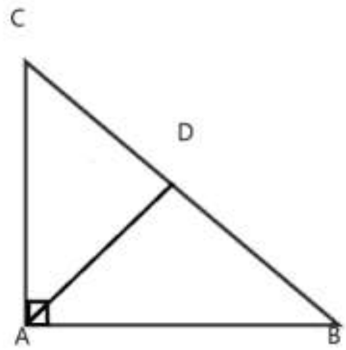
4. Prove in as many ways as you can that the median AD of a right triangle ABC (CD=DB) equals half the hypotenuse CB.

5.

Data given	Prove that:
ABC right triangle	AD = ½ BC
AD is the median	
CD=DB	

6.

Data given	Prove that:
ABC right triangle	AD = ½ BC
AD is the median	
CD=DB	



In previous research studies, the geometrical MSTs that participants were asked to solve were accompanied by the relevant geometrical figure. It is difficult to organize a research among students and use all six alternative types of presentation of a geometrical task. The reason is first of all that such a study would require that each alternative task is chosen with specific cognitive and figure apprehension criteria. Secondly, each task would need to be solved by a large number of subjects in order to achieve representative results. As a first step, in this study, half of the tasks were accompanied by the relevant figure and half of the tasks were not. Duval (2006) has distinguished among three different cognitive functions that students experience in their effort to understand and solve geometrical problems; (a) visual process, (b) construction of figures and (c) discursive process for exploring, explaining or proving. When the presentation of a task does not include the relevant figure, students have to construct it in order to find a solution. This acts as an extra difficulty to the task solution which has been recognized by the authors of Mathematics textbooks and it is for this reason that they propose that geometrical tasks include both registers; the natural language and the graphical register. In our study, Tasks 1 and 3 include both registers and thus required the second level of visualization as explained earlier for their solution. On the contrary, Tasks 2 and 4 were presented in one register only, i.e. the natural language and therefore both levels of visualization were necessary for their solution.

Population and Data Collection

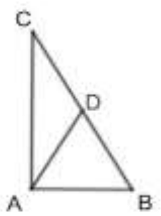
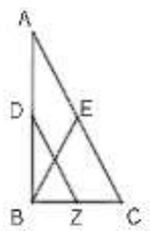
A written test was administered to 147 eleventh grade students (77 boys and 72 girls from 7 classes in 5 Greek schools). All 147 students completed the test in usual classroom conditions. The students were randomly selected from 5 urban schools in Greece. Students of various socio-cultural environments were included in the sample. The test consisted of four geometry tasks (Figure 1) and students were explicitly asked to solve them in as many ways as possible. The reason for choosing the area of Geometry is first of all that geometry is closely linked to visualization and secondly, that this area of mathematics provides a rich environment of MSTs. As Levav-Waynberg and Leikin (2009) have stated, almost every geometry problem in school textbooks can be adapted so as to satisfy the characteristics of an MST. The tasks have been used by other researchers as well. Task 1 is used by Leikin (2011) and Task 2 by Levav-Waynberg and Leikin (2012a) and Gridos, Avgerinos, Mamona-Downs & Vlachou (2021). All the tasks are similar to tasks included in Grade 9 and 10 Mathematics textbooks. In Tasks 1 and 3 the wording of the task was accompanied by the relevant figure, while in Tasks 2 and 4 the wording was not accompanied by the geometrical figure. The specific tasks were chosen because they symbolize different representations, i.e. different geometrical figures, which refer to specific properties that are taught through classical Euclidean geometry in schools in Greece, a part of the syllabus taught to this specific age group of students.

Data Analysis

In order to investigate how the presentation of a geometrical task can influence student creativity, we calculate the creativity components of fluency, flexibility and originality for each solution used by the students. In order to do this, we first identify the number of different solutions and we also investigate the use of auxiliary lines by students. For this analysis, we used both descriptive statistics and Leikin’s (2009) scoring scheme for mathematical creativity. Next and in order to investigate the relations among student creativity, visualization and geometrical figure apprehension we used the following statistical tools; t-test at 95% significance level and multiple regression analysis with computer software SPSS. A series of multiple regression analyses were performed with creativity (and each of its components) as the dependent variables and solution of a task with or without the geometrical figure as the independent variable. Multiple regression analysis allowed us to characterize these relationships not only in terms of their statistical significance but also in terms of their strength, through specific measures, including regression coefficients and proportion of variance (Cohen, Cohen, West, & Aiken, 2003). Finally, an implicative statistical analysis was conducted using the computer software C.H.I.C. The implicative method enables the observation of the birth of an ability according to the stages of genetic psychology as well as spotting of the unchanged or stable way of thinking of the subjects. The relations to which we conclude are not relations of causation. In other words, the implicative method helps us conclude that success in a task entails the success in another task with which the first task is related. Similarly, failure in one task entails failure in another task with which the first task is related (Gras, Suzuki, Guillet, & Spagnolo, 2008). On the implicative diagram we symbolize with Ti, i=1, 2, 3, 4 the task referred to and symbolize with Sj, j=0, 1, 2, 3, 4 the number of solutions for each task.

Figure 1

The MSTs That Were Included in The Test

Task 1: Prove in as many ways you can that, that the median drawn to the hypotenuse of the right triangle equals half the hypotenuse. 	*Task 3:* Let a rectangular triangle ABC (B = 90°), D midpoint of segment AB, E the midpoint of segment AC and Z the midpoint of segment BC. Prove in as many ways you can that DZ = EB. 
Task 2: AB is a diameter on a circle with center O. D and E are points on circle O so that DO//EB. C is the intersection point of AD and BE. Prove in as many ways as you can that CB=AB.	*Task 4:* Find the center of a circle in as many ways as you can, if you only know the circumference of the circle.

Results

We first analyzed the number of solutions produced by students for each Task. Table 1 below, shows the percentages of students who managed to solve each Task in one, two or more than two ways. In other words, Table 1 shows students' fluency per Task.

Table 1

Overall Fluency of Students Per Task

		% of students			
		Task 1	Task 2	Task 3	Task 4
Number of Solutions	None	24.5	41.5	25.9	43.5
	One	47.6	49	45.6	40.1
	Two	23.8	8.2	22.4	12.9
	Three and above	4.1	1.3	6.1	2.7

So, for example, as far as students' fluency in Task 1 is concerned 24.5% of the students could not solve Task 1, almost half of the students found one solution, 23.8% solved it in two ways, 3.4% of the students produced three solutions and 0.7% of the students managed to solve it in four ways. This means that only 4.1% of the student population participating in the test solved the task in more than two ways. Similarly, in the other Tasks the majority of students produced one solution (apart from Task 4 where the majority of students did not provide any solution) and a very low minority produced more than two solutions.

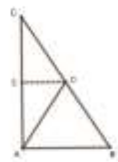
As explained in the previous sections, in order to calculate the flexibility component, we categorized students' solutions in collective solution spaces. For Task 1, we identified five collective solution spaces, for Tasks 2 and 3 we identified three solution spaces and finally, for Task 4, we identified four solution spaces. In order to explain how we categorized student solutions we give a detailed example with Task 1 in Figure 2. Then, in Figure 3, we briefly present the collective solution spaces identified in Tasks 2, 3 and 4.

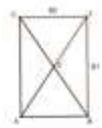
Figure 2 presents the five collective solution spaces identified and also, the percentages of students that used each specific solution for Task 1. This Figure helps us observe the flexibility of the group of students. Students produced ten different solutions that correspond to five solution spaces. Most students (39.2%) used the solution that involves the comparison of triangles (T1com1, T1com2). Students constructed the median of the triangle joining the middle of the sides and then compared the two triangles. The proof that the school textbook presents is based on the construction of the median and the observation that it is an altitude as well (T1med1). It is remarkable that this solution was used by fewer students (27.1%). Even though students used the same auxiliary construction - the median - our findings indicate that students are more familiar with triangle congruence. This is not however the case as far as triangle similarity is concerned as only 6.2% of the students used it to prove the sentence. This indicates that students face difficulties in recognizing the proportionality embedded in the triangles which arose as a result of figure modification.

All the solutions provided by students required the construction of auxiliary lines. In fact, as far as auxiliary lines are concerned, solutions are distinguished in two broader categories in terms of the final configuration of the figure. In the first and fourth solution spaces, the auxiliary lines create sub-figures within the given figure. In the second, third and fifth solution spaces the construction of the auxiliary lines results in the construction of figures of which the given figure is a part. The percentage of the students that use the first category of auxiliary lines is higher than the second one indicating that the latter figural processing is much more demanding. It is evident that a geometrical figure can be modified in many ways, each of which can lead the student to a different solution of the task. Nevertheless, some students constructed the same auxiliary line, but solved the task differently. These students apprehend operatively the figure in the same way but have different discursive apprehension as, in order to find the solutions, they used different properties and theories for the same auxiliary structure.

Figure 2

Collective Solution Spaces for Task 1

First solution space	<u>Solution 1^a</u> (T1med1) 23% The median DM of triangle ADB is constructed. The line segment DM connects the midpoints of the two sides of the ABC triangle. Thus DM // AC. Knowing that AC AB, then DM AB DM is the median but also, the altitude of triangle ADB and thus, triangle ADB is isosceles. Therefore, AD=DB=BC/2	
	<u>Solution 1^b</u> (T1med2) 4,1% The median DE of triangle ADC is constructed. The line segment DE connects the midpoints of the two sides of the ABC triangle. Thus DE // AB. Knowing that AB AC, then DE AC DM is the median of ADC triangle and DM is also the altitude of triangle ADC. Therefore, ADC triangle isosceles and AD=DB=BC/2	
	<u>Solution 1^c</u> (T1com1) 35,1% The median DM of triangle ADB is constructed. The line segment DM connects the midpoints of the two sides of the ABC triangle. Thus DM // AC. Knowing that AC AB, then DM AB Comparing the triangles AMD and BMD: • DMis a common side • The angles DMA and DMB are equal (900). • AM=MB By using the Side-Angle-Side rule, the two triangles are congruent. Therefore, AD=DB=BC/2	
	<u>Solution 1^d</u> (T1com2) 4,1% The median DE of triangle ADC is constructed. The line segment DE connects the midpoints of the two sides of the ABC triangle. Thus DE // AB. Knowing that AB AC, then DE AC Comparing the triangles DEA and DEC: • DE is a common side • The angles DEC and DEA are equal (900). • AE=CE The two triangles are congruent according to the Side-Angle-Side rule. Therefore, AD=DB=BC/2	
Second solution space	<u>Solution 2^a</u> (T1cir) 4,1 % The angle BAC = 90 ° The right-angle triangle ABC is inscribed in a circle. The hypotenuse BC of triangle ABC is also the diameter of the circle and D its centre. Therefore, AD=DB=DC=BC/2	
Third solution space	<u>Solution 3^a</u> (T1dom) 14,2% Extend AD to D so that AD = DZ. Join C and Z. Join Z and B. Diagonals bisect each other. Thus, ABZC is a parallelogram. The angle BAC=90o. Thus, ABZΓ is a rectangle. AZ=BC (diagonals of a rectangle) Thus, AD=DZ=DB=DC=BC/2	

	<u>Solution 3^b</u> (T1dot) 13,5%
	From the vertex B a parallel line to the AC is constructed (e1)//AC
	From the vertex C a parallel line to AB is constructed (e2)//AB.
	Let Z be the intersection of e1 and e2.
	ABZC is a rectangle as the angle BAC=90o.
	AZ=BC (diagonals of a rectangle)
	Thus, AD=DZ=DB=DC=BC/2
		

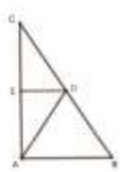
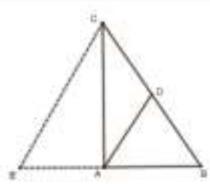
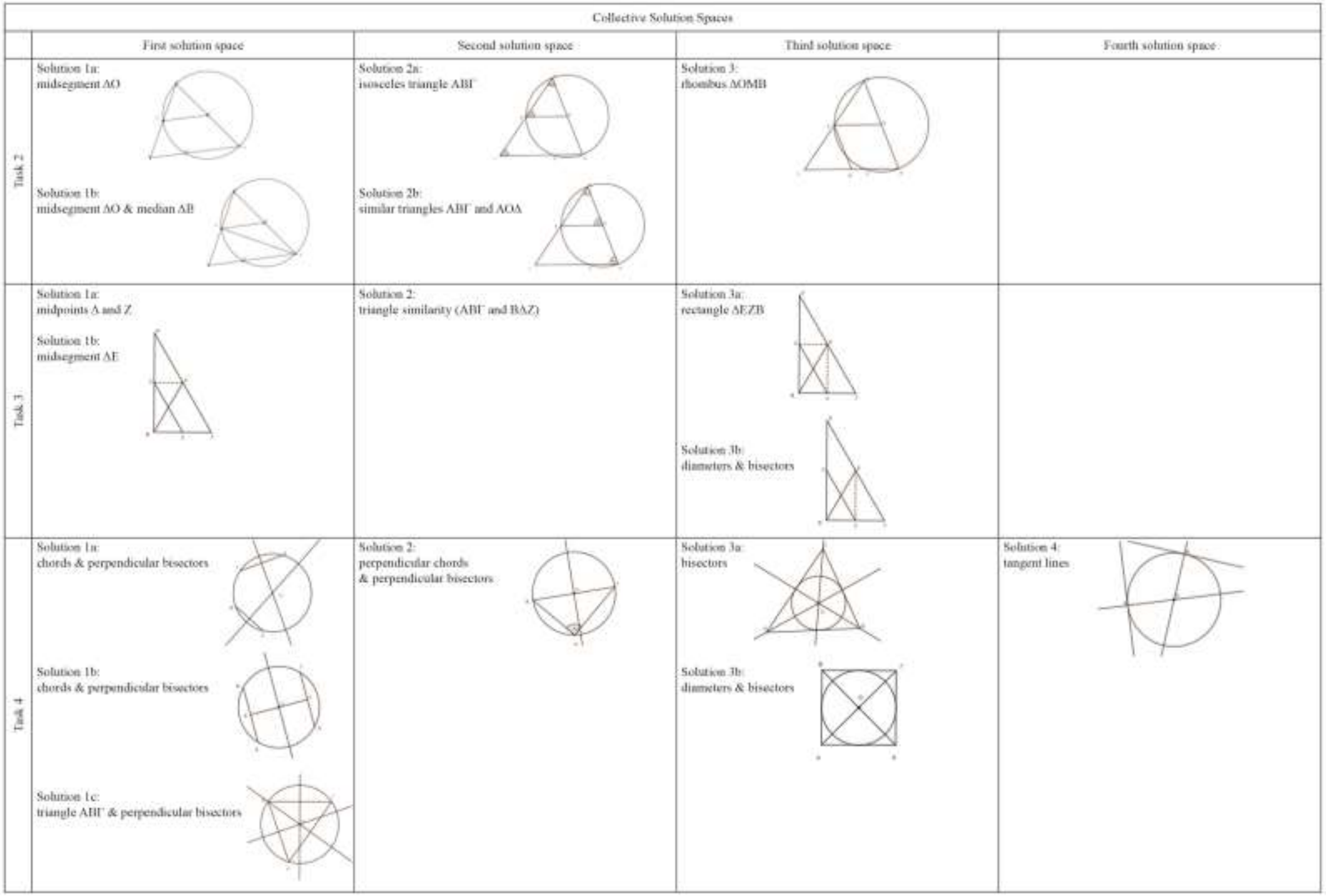
Fourth solution space	<u>Solution 4^a</u> (T1sim1) 4,7% Median DM of triangle ADB is constructed. The DM joins the midpoints of two sides. Thus DM // = 1 / 2AC Comparing the triangles ABCand DMA: • The angles BAC = BMD = 90° • MA=1/2AB • DM=1/2AC The triangles ABC and DMA are similar (with ratio 1:2) as according to the Side-Angle-Side rule two sides of one triangle (ABC) are in the same proportion to the corresponding sides of the other (DMA) and the included angles are equal. Therefore, AD=1/2BC	
	<u>Solution 4^b</u> (T1sim2) 1,4% Median DE of triangle ADC is constructed. The DE joins the midpoints of two sides. Thus DE // = 1 / 2AB. Comparing the triangles ABC and AED: • The angles BAC = DEC =90o • ED=1/2AB • EA=1/2AC The triangles ABC and AED are similar (with ratio 1:2) as according to the SAS rule two sides of one triangle (ABC) are in the same proportion to the corresponding sides of the other (AED) and the included angles are equal. Therefore, AD=1/2BC	
Fifth solution space	Solution 5 (T1ext) 2% SegmentAB΄ that is equal with the side AB Is constructed D is the midpoint of BC and A the midpoint of B΄B thus DA//=1/2B΄C (1) Comparing the triangles B΄CA and BAC: • CAB= CAB΄=90° • CA common side • B΄A=BA The two triangles BAC=BAC are congruent according to the Side-Angle-Side rule. Thus B΄C=BC and B΄CB isosceles (2) Given (1), (2) AD=B΄C/2=BC/2	

Figure 3

Collective solution spaces for Tasks 2, 3 and 4.



Having analysed student solutions and having identified the solutions spaces for each Task, the creativity components of fluency, flexibility and originality were calculated for each Task separately. Total creativity was calculated too. First of all, the Cronbach's Alpha reliability coefficient for the variables examined which are fluency, flexibility and originality is 0.738, 0.754 and 0.781, respectively, suggesting that the tasks have relatively high internal consistency. Furthermore, all tasks were content and face validated by two experienced secondary school mathematics teachers and two mathematics education professors.

The solutions provided by students were classified according to the strategy they used. Then, the three creativity components - fluency, flexibility, originality - as well as total creativity were calculated based on the scoring scheme that Leikin (2009) proposed for the evaluation of performance in MSTs. So, fluency and flexibility reflect the number of appropriate and different solutions produced respectively. Leikin (2009) uses a specific score scheme for calculating flexibility which is the following; 10 for the first solution, 10 for an additional solution but from a different solution space, 1 for a solution which belongs to a solution space previously used but differs in strategy and 0.1 for a solution which is almost the same with a strategy already used. Finally, originality is scored by Leikin (2009) with 10, 1 and 0.1 depending on the number of students who used a specific strategy. If less than 15% of students have used a strategy then originality is given a score of 10. If more than 40% of students have used a specific strategy then originality is given a score of 0.1. If a percentage between 15 and 40% of students have used a specific strategy then originality is given a score of 1. In other words, originality was evaluated on the rate of occurrence of each solution for all students. So for example, in Task 1 and for the first solution space, each solution is rated with 0.1, for the third solution space it is rated with 1, while for the second, fourth and fifth solutions spaces solutions are rated with 10. The highest originality score reached 21, which means that no student found all the innovative solutions. The student who attained the highest originality score produced solutions that involved the construction of figures embracing the given figure (T1cir, T1dot, T1ext). As an example, Table 2 presents the scores for the fluency, flexibility, originality and total creativity in Task 1. Similar calculations for the creativity components as well as for total creativity have been performed for Tasks 2, 3 and 4. It is worth noting here that Task 1 was calculated with the highest total creativity score among all four Tasks.

In MSTs that the wording is accompanied by the relevant figure (e.g. Task 1), second level visualization is needed in order to interpret the geometric figure and operative figure apprehension is needed in order to perform figure modifications. In MSTs in which the relevant figure is not given (e.g. Task 4), first level of visualization is also essential for the construction of the geometric figure. When a student solves tasks in this latter category, it means that s/he has developed the corresponding visualization abilities. Another way to check whether students have developed the necessary visualization abilities is to look at the percentages of correct solutions in each category. For each category of tasks, a new variable is created relating to the ability to solve the specific tasks depending on whether the relevant figure is given.

Table 2

Creativity component and Total Creativity Scores for Task 1

Solutions	Fluency	Flexibility Flx_i	Originality Or_i	Total creativity $Cr_i = Flx_i * Or_i$
T1med1	1	10	0.1	1
T1med2	1	0.1	0.1	0.01
T1com1	1	1	0.1	0.1
T1com2	1	0.1	0.1	0.01
T1cir	1	10	10	100
T1dom	1	10	1	10
T1dot	1	1	1	1
T1sim1	1	10	10	100
T1sim2	1	1	10	10
T1ext	1	10	10	100
Total	10	53.2	42.4	322.12

When the figure is given, the second level of visualization ability can be assessed, which specifically refers to the ability to interpret the geometric figure and to modify it. When the figure is not given, the ability to construct and modify a geometric figure in order to produce a solution can be assessed. Before we move on with the investigation of the relations among student creativity, visualization and geometrical figure apprehension, Figure 4 presents an example of one student solving Task 4 (MST without the given figure) in six different ways. In this example, one can observe the student using both levels of visualization ie. the conversion of the verbal description into the construction of the figure and then, the transition from the geometrical figure to the solution of the MST, where both the operative and discursive apprehension of the figure are involved.

The next step in our investigation was to study in more depth the influence of the presentation of the Task by comparing the creativity component scores among the two categories of Tasks. Therefore, Table 3 presents the descriptive data for students' fluency, flexibility, originality and total creativity in two categories of tasks depending on whether the wording of the task is accompanied by the relevant figure or not.

Figure 4

Solutions provided by one student to Task 4

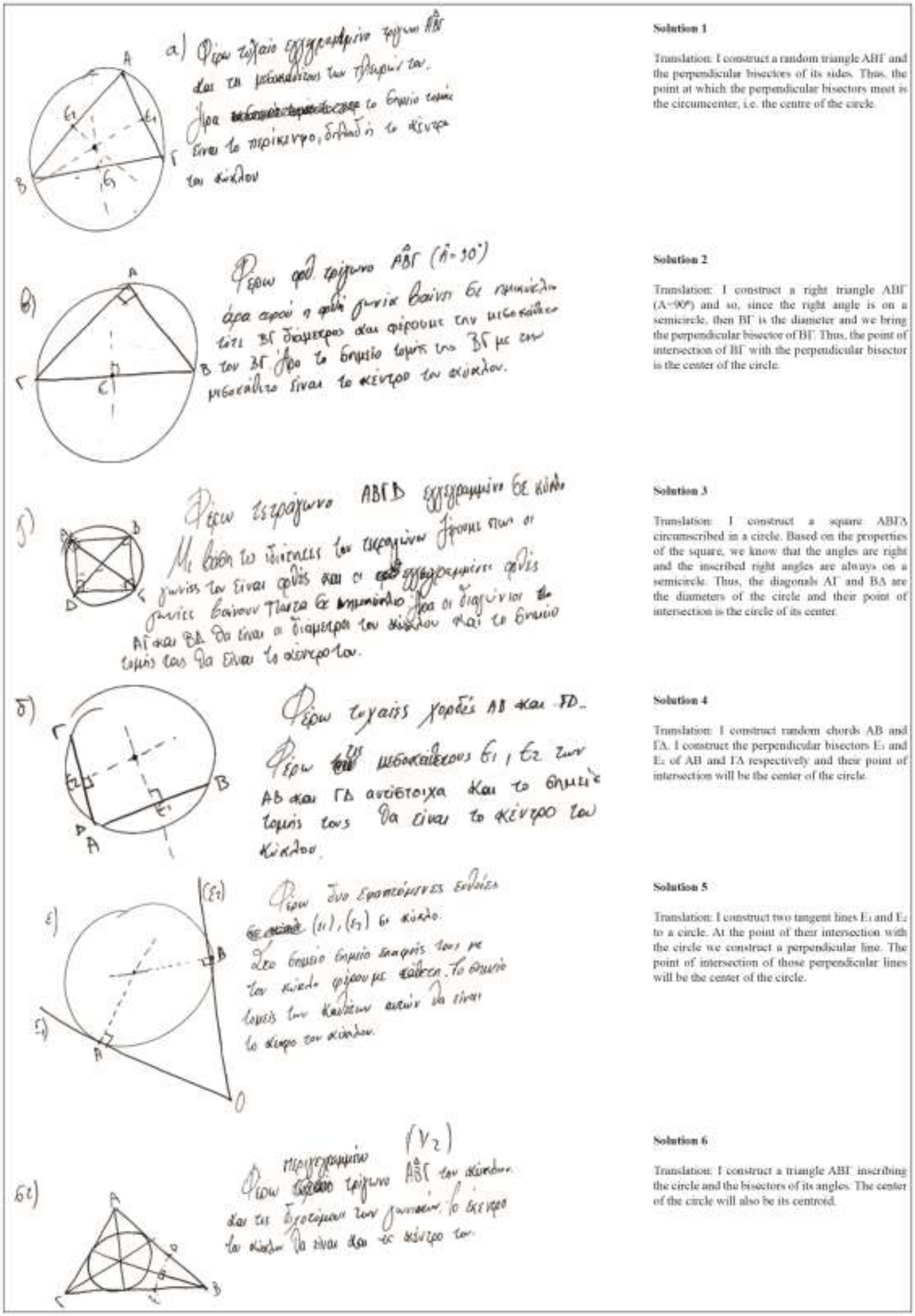


Table 3

Descriptive data for students' creativity

	Tasks with figure		Tasks without figure	
	x̄	SD	x̄	SD
Fluency	2.160	1.422	1.442	1.205
Flexibility	19.881	12.651	13.930	11.370
Originality	2.891	5.001	3.925	5.989
Total Creativity	58.701	121.932	67.274	136.483

On the basis of the results of the t-test, the number of solutions students give to Tasks 1 and 3 (M = 2.16, SD = 1.42) (i.e. the wording of the task is accompanied by the relevant figure) is higher compared to Tasks 2 and 4 (M = 1.44, SD = 1.21) (i.e. tasks in which the figures are not given) and the difference is statistically significant (t = 6.02, df = 146, p <0.001). Concerning flexibility of the solutions to the tasks that the figure is given (M = 19.88, SD = 12.65) and the tasks without the corresponding figure (M = 13.93, SD = 11.37), significant differences are indicated with an apparently higher score in the tasks in which the figure is given (t = 5.57, df = 146, p < 0.001). Despite the fact that our results show that students' fluency and flexibility is higher in tasks in which the figure is given, this is not the case with the originality of the solutions (t = -1.87, df = 146, p > 0.05) and total creativity (t = -0.71, df = 146, p > 0.05). In fact, there are no significant differences as far as originality and creativity are concerned between the two types of tasks.

A series of multiple regression analyses were used to assess the relationship between the two factors of interest in this study, namely, solution of a task with or without a geometrical figure, which are the independent variables, and each of the dependent variables, namely creativity and each of its components (fluency, flexibility and originality). As explained in the Data analysis section, through multiple regression analysis we were able to characterize these relationships both in terms of their statistical significance and their strength. It can therefore be inferred which of the two types of tasks contributes more to mathematical creativity. Statistically significant prediction models were obtained for each dependent variable using the Enter method (Fluency: F = 14.467, p = 0.022; Flexibility: F = 13.121, p = 0.034; Originality: F = 16.352, p = 0.017 and Total Creativity: F = 19.032, p = 0.001). Table 4 presents the results of the multiple regression analyses.

Table 4

Multiple regression results regarding the relationship between the solution of tasks with or without figure and mathematical creativity components

		Tasks with the relevant figure		Tasks without the relevant figure	
		B (SE)	*β*	*B* (SE)	*β*
Mathematical Creativity	Fluency	0.10 (0.03)	0.289**	-0.01 (0.01)	-0.017
	Flexibility	0.05 (0.03)	0.164	0.02 (0.04)	0.072*
	Originality	-0.03 (0.02)	-0.098**	0.08 (0.02)	0.252*
	Total Creativity	0.04 (0.03)	.104	0.12 (0.02)	0.344**

R^2 = 0.145 for fluency, R^2 = 0.125 for flexibility, R^2 = 0.204 for originality, R^2 = 0.116 for total creativity, * $p < 0.05$, ** $p < 0.01$

Solving a task with a geometrical figure is significantly related with students' fluency and originality (β = 0.289, p = 0.002 and β = -0.098, p = 0.004 respectively) and it can explain 15% and 20% of the variance in the total performance of the fluency and originality, respectively. Solving a task without a geometrical figure is significantly related with students' flexibility (β = 0.072, p = 0.011), originality (β = 0.252, p = 0.026) and total creativity (β = 0.344, p = 0.001) and it can explain 13%, 20% and 12% of the variance in the total performance of flexibility, originality and total creativity respectively. The beta values show that tasks that are not accompanied by the geometrical figure have a positive effect on students' originality, while tasks that are accompanied by the geometrical figure have a negative impact on it.

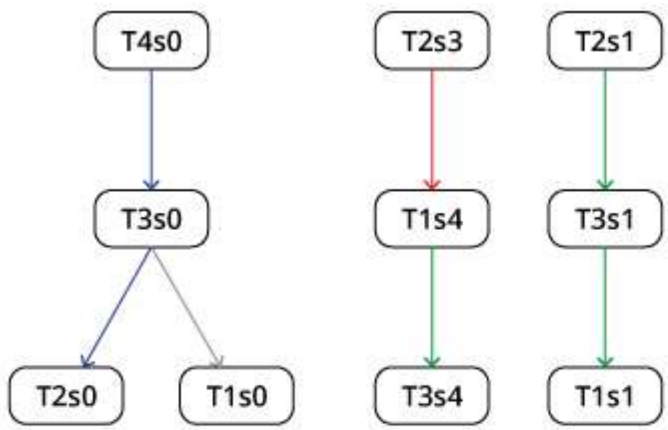
Finally, we applied the implicative statistical analysis of R. Gras (Gras, Suzuki, Guillet, & Spagnolo, 2008). In the following implicative diagram, implications at a significance level of 99% (red), 95% (blue), 90% (green) and 80% (grey) are indicated.

Three independent implicative chains are distinguished at the implicative diagram:

- The first chain (T4s0 →T3s0 →T2s0, T1s0) includes the cases in which no solution was given
- The second chain (T2s3 →T1s4 →T3s4) includes the variables in which three or four solutions were given
- The third chain (T2s1 →T3s1 →T1s1) includes the variables in which one solution was given

Figure 5

Implicative diagram of students' number of solutions



According to the implicative diagram, creativity that is indicated by the number of solutions appears to have a more powerful effect than figure apprehension in the creation of implicative relationships. These implicative relationships are mainly created between responses with the same number of solutions rather than between responses to the same category of problems in terms of the presence or not of a geometrical figure. Although the three chains are separated because of the limited number of variables involved we could not argue that we have the phenomenon of "compartmentalization". It is also worth mentioning here that the chains have as a starting point a task that is not accompanied by the figure, either Task 2 or 4. This finding indicates that the success in a task that the wording is not accompanied by the relevant figure entails the success in tasks in which the wording is accompanied by the relevant figure.

Discussion

This paper aims to enrich the existing literature regarding high school students' mathematical creativity in geometry when solving MSTs. For this purpose, the relationship of mathematical creativity with both visualization and geometrical figure apprehension was examined. In tasks whose wording is not accompanied by the relevant figure, two levels of visualization are needed. The first one involves visualizing that enables the sketch to be produced based on the problem's verbal description and producing the sketch. Thus, discursive apprehension of the figure is required, leading to the geometric construction, which here is conceived both as the mental process of making the figure and as the actual production of the sketch. The second level of visualization has to do with the interpretation and mental process of operating on the figure in order to solve the geometrical MST. Thus, operative figure apprehension is essential as the students modify the figure and produce different configuration(s) in order to provide solutions. Overall, students' actual product (sketch produced) draws on their geometric thinking which involves the discursive process and the visual process of making the sketch (Visualization level 1), and leads to the visual process of operating on the sketch (Visualization level 2). In tasks whose wording is accompanied by the relevant figure, the visual process for their solution involves the second level of visualization only.

Another important finding relates to the way students modify a geometrical figure and has to do with the construction of auxiliary lines when the relevant figure is given. Our results showed that more students constructed auxiliary lines inside and not outside the given geometrical figure. Additionally, a surprising result was that although some students constructed the same auxiliary line, they then followed a different path to the solution of the task. This proves that different students can have similar operational but different discursive apprehension of a figure. Palatnik and Dreyfus (2018) pointed out that by introducing auxiliary lines students can recall known facts or definitions and modify the given diagrams. This way they anticipate to receive helpful information which will lead them to the solution or to the recognition of an already learned procedure. However, the use of auxiliary lines is a source of difficulty for students. According to Hsu (2007), in order to construct an auxiliary line and therefore visualize a solution, a student needs to approach the figure dynamically and apply transformational observation. This is probably the reason why, for example, a quarter of the students involved in this study did not solve Task 1. Our results confirmed that in order to produce solutions to a task, apart from perceptual, sequential and discursive apprehension, operative apprehension is also required, since students must construct auxiliary lines. This is in line with Duval's (1995) point of view that many different apprehension processes are necessary at the same time in order for students to understand a mathematical figures. In summary, our findings suggest that the auxiliary lines are divided into two types: (a) the auxiliary lines that create sub-figures within the given figure and (b) the construction of the auxiliary lines which results in the construction of figures of which the given figure is a part. According to the percentages of student solutions, the latter figural processing is much more demanding for students as it requires increased abstract thinking and the ability to visualize the result of the figure modification. Duval (1999) has stated that is the meteorologic modification of the geometrical figure which gives students the ability to perceive the figure both as a whole and also, recognize the subfigures within it. This is a skill needed in order for students to be able to solve a task in more than one way. It is through the meteorologic modification that students can focus on different figures at the same time, recognize new properties, construct new elements in the shape and discover different solution strategies (Michael-Chrysanthou & Gagatsis, 2015). This finding shed light to an effective strategy through which students can solve geometry problems. Such a strategy could include the use of tasks that aim to overcome students' perceptual recognition of a figure in order to lead to operational apprehension through meteorologic modification. This has been the subject of a study by Tzefriou, Santorinaiou, Deliyianni & Elia (2020) who have highlighted the role of mereological modifications in supporting students to breach the didactical contract. Their study concluded that students faced more difficulties with the operative apprehension of figures compared to the other apprehensions. The authors of this study, suggested that students should be more exposed to MSTs in order to help them breach the didactic contract and discover new ideas and solutions. As Brousseau (1984) has stated, learning takes place when the didactic contract is breached.

When investigating the relations among creativity and the presentation of a geometrical task, the type of creativity students demonstrate given an MST that may or may not be accompanied by the relevant figure shows variations. To be more precise, when the relevant figure is given, although fluency and flexibility are higher, originality is not influenced neither positively nor negatively. In tasks that are presented only verbally and, thus, students must construct the geometrical figure in order to solve them, originality is higher. In fact, the presence of the figure in MSTs seems to work negatively in terms of finding original solutions. In summary, the results show that students' total creativity is higher in tasks in which the relevant figure is not given. We therefore suggest that the inclusion of figure constructions in teaching and learning will significantly enhance the development of students' creativity and the ability to solve MSTs.
Another important finding of our study was that, when compared to spatial ability, the ability of a student to produce multiple solutions is a more powerful predictor for the ability of a student to solve similar problems in multiple ways.

The results of our study are similar to the results of a study by Gridos et al. (2021), who investigated the ways in which students perceive and process a geometrical figure and their relation to mathematical creativity. It is important to note here that, as stated earlier in this article, one of the tasks used in the study by Gridos et al. (2021) was also used in our study (Task 2). Gridos et al. (2021) deduced that it is the operational apprehension of geometrical figures and not the perceptual which positively predicts students' fluency, flexibility and originality.

In conclusion, it seems that the presentation of a geometrical task and more specifically, the presence of the relevant geometrical figure in the wording of a task indeed influences the ability of students to produce more than one solutions. Consequently, student creativity is also influenced. The reasons for this relation is explained by two general arguments; (a) there are various different cognitive processes involved in the solution of a geometrical task and (b) students apprehend and modify figures in various different ways.

The sample of students as well as the chosen tools and tasks for this study are possible limitations and therefore, more studies should be performed on this topic. By differentiating the geometrical tasks or choosing a different age group of students, researchers could contribute to the existing literature related to student creativity in geometry. Furthermore, the implications of these findings should be investigated from the learning and teaching perspective. How can we use this knowledge in the teaching of geometry? What do these findings imply for mathematics curricula? Future research could focus on studying the effects of intervention programs aiming to improve, as a first step, students' geometrical figure apprehension (Michael, Gagatsis, Avgerinos, & Kuzniak, 2011; Gagatsis, 2015; Gagatsis, Michael–Chrysanthou, Deliyianni,, Panaoura, & Papagiannis, 2015) so that the geometrical figure functions as a heuristic tool in task solving and not as an obstacle (Michael –

Chrysanthou, & Gagatsis, 2013). As a second step, we could examine the effects of an intervention program aiming to teach figure construction skills to students. Finally, another important contribution to the existing literature on student mathematical creativity will be that of Gagatsis and Geitona (2021) who will attempt a multidimensional investigation of student creativity by taking into consideration students' spatial abilities and the ability to apprehend geometrical figures through multiple-solution geometrical problems.

References

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics, 52*(3), 215–241.

Bicer, A. (2021). Multiple representations and mathematical creativity. *Thinking Skills and Creativity*, *Vol. 42*, https://10.1016/j.tsc.2021.100960

Brousseau, G. (1984). The crucial role of the didactical contract in the analysis and construction of situations in teaching and learning mathematics. In H. G. Steiner (Ed.), *Theory of mathematics education* (pp. 110-119). Belfield, Germany: IDMUB.

Carroll, J.B. (1993). *Human cognitive abilities: A survey of factor-analytical studies.* United Kingdom: Cambridge University Press.

Chiu, M. S. (2009). Approaches to the teaching of creative and non-creative mathematical problems. *International Journal of Science and Mathematics Education*, *7*, 55-79.

Cohen, J., Cohen, P., West, S., & Aiken, L. (2003). *Applied multiple regression/correlation analysis for the behavioral sciences*. (3rd ed.). Mahwah, NJ: Lawrence.

Confrey, J., & Smith, E. (1991). A framework for functions: Prototypes, multiple representations and transformations. In R. G. Underhill (Ed.), *Proceedings of the 13th annual meeting of the North American Chapter of The International Group for the Psychology of Mathematics Education* (pp. 57-63). Blacksburg: Virginia Polytechnic Institute and State University.

Dindyal, J. (2015). Geometry in the early years: a commentary. *ZDM Mathematics Education*, *47*(3), 519–529.

Duval, R. (1995). Geometrical Pictures: Kinds of Representation and Specific Processings. In R. Sutherland & J. Mason (Eds.), *Exploiting Mental Imagery with Computers in Mathematics Education,* (p.142-157). Germany: Springer.

Duval, R. (1999). Representations, vision and visualization: Cognitive functions in mathematical thinking. Basic issues for learning. In F. Hitt & M. Santos (Eds.), *Proceedings of the 21st Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education. (Vol. 1,* pp. 3–26). Morelos, Mexico.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics, 61*(1), 103–131.

Duval, R. (2014). The first crucial point in geometry learning: Visualization. *Mediterranean Journal for Research in Mathematics Education*, *13*(1-2), 1-28.

Elia., I., Gagatsis, A., & Gras, R. (2005). Can we “trace” the phenomenon of compartmentalization by using the I.S.A.? An application for the concept of function. In R. Gras, F. Spagnolo & J. David (Eds.), *Proceedings of the Third International Conference I.S.A. Implicative Statistic Analysis* (pp. 175-185). Palermo, Italy: Universita degli Studi di Palermo.

Elia, I., Gagatsis, A., & Demetriou, A. (2007). The effects of different modes of representation on the solution of one-step additive problems. *Learning and Instruction, 17,* 658-672.

Elia, I., Panaoura, A., Eracleous, A., & Gagatsis, A. (2007). Relations between secondary pupils’ conceptions about functions and problem solving in different representations. *International Journal of Science and Mathematics Education, 5,* 533-556.

Elliot, J. & Smith, I.M. (1983). *An international dictionary of spatial tests.* Windsor, United Kingdom: The NFER-Nelson Publishing Company, Ltd.

Ervynck, G. (1991). Mathematical creativity. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 42–53). Dordrecht, Netherlands: Kluwer.

Gagatsis, A. (2015). Explorando el rol de las figuras geométricas en el pensamiento geométrico. In B. D’Amore & M.I. Fandiño Pinilla (Eds), *Didáctica de la Matemática - Una mirada internacional, empírica y teórica* (pp. 231-248). Chia: Universidad de la Sabana.

Gagatsis, A., & Geitona Z. (2021). A multidimensional approach to students’ creativity in geometry: spatial ability, geometrical figure apprehension and multiple solutions in geometrical problems. *Mediterranean Journal for Research in Mathematics Education, Vol. 18*, 5-16, 2021.

Gagatsis, A. & Kalogirou, P. (2013). *Development of spatial ability and geometric figure apprehension.* Nicosia: University of Cyprus (in Greek).

Gagatsis, A. & Shiakalli, M. (2004). Ability to translate from one representation of the concept of function to another and mathematical problem solving. *Educational Psychology, 24*(5), 645-657.

Gagatsis, A., Christodoulou, T. & Elia, I. (2013). How young students deal with recognition and conversion tasks of graphs. *Acta Didactica Universitatis Comenianae – Mathematics, 13,* 1-16.

Gagatsis, A., Deliyianni, E., Elia, I., Panaoura, A., & Michael-Chrysanthou, P. (2016). Fostering Representational Flexibility in the Mathematical Working Space of Rational Numbers. *Bolema: Boletim de Educação Matemática, 30*(54), 287.

Gagatsis, A., Michael – Chrysanthou, P., Deliyianni, E., Panaoura, A., & Papagiannis, C. (2015). An insight to students’ geometrical figure apprehension through the context of the fundamental educational thought. *Communication & Cognition, 48*(3-4), 89-128.

Gras R., Suzuki E., Guillet F., & Spagnolo F. (2008). *Statistical implicative analysis.* Germany: Springer.

Gridos, P., Gagatsis, A., Deliyianni, E., Elia, I., & Samartzis, P. (2018). The relation between spatial ability and ability to solve with multiple ways in geometry. *Paper presented in: ISSC 2018 - International Conference on Logics of image: Visual Learning, Logic and Philosophy of Form in East and West, Crete, Greece. Research Gate.*

Gridos, P., Avgerinos, E., Mamona-Downs, J. & Vlachou, R. (2021). Geometrical Figure Apprehension, Construction of Auxiliary Lines, and Multiple Solutions in Problem Solving: Aspects of Mathematical Creativity in School

Geometry. *International Journal of Science and Mathematics Education.* https://doi.org/10.1007/s10763-021-10155-4

Gridos, P., Avgerinos, E., Deliyianni, E., Elia, I., Gagatsis, A., Geitona, Z. (2021). Unpacking The Relation Between Spatial Abilities and Creativity in Geometry. *The European Educational Researcher, 4*(3), 307-328. https://doi.org/10.31757/euer.433

Guilford, J. P. (1967). *The nature of human intelligence.* New York: McGraw-Hill.

Gutiérrez, A. (1996). Visualization in 3-dimensional geometry: In search of a framework. In L. Puig & A. Guttierez (Eds.), *Proceedings of the 20th conference of the International Group for the Psychology of Mathematics Education* (*Vol*. 1, pp. 3–19). Valencia: Universidad de Valencia.

Halpern, D.F. (2000). *Sex differences and cognitive abilities.* Mahwah, NJ: Erlbaum.

Haylock, D. W. (1987). A framework for assessing mathematical creativity in school children. *Education Studies in Mathematics*, *18*(1), 59–74.

Hiebert, J. & Carpenter, T. P. (1992). Learning and teaching with understaning. In. D. A. Ggrouws (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 65-97). New York, NY: Macmillan.

Hsu H. (2007). Geometric calculations are more than calculations. In J.H Woo, H.C Lew, K.S Park, D.Y Seo (Eds.) *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education* (*Vol*. 3, pp. 57–64). Seoul, Korea.

Kaufman, J. C., & Beghetto, R. A. (2009). Beyond big and little: The four C model of creativity. *Review of General Psychology,* 13, 1-12.

Krutetskii, V. A. (1976). *The Psychology of Mathematical Abilities in Schoolchildren.* Chicago: University of Chicago Press.

Leikin, R. (2007). Habits of mind associated with advanced mathematical thinking and solution spaces of mathematical tasks. In D. Pitta-Pantazi & G. Philippou (Eds.), *Proceedings of the Fifth Conference of the European Society for Research in Mathematics Education* (pp. 2330-2339). Larnaka, Cyprus: University of Cyprus and ERME.

Leikin, R. (2009). Exploring mathematical creativity using multiple solution tasks. In R. Leikin, A. Berman and B. Koichu (Eds.), *Creativity in mathematics and the education of gifted students* (pp. 129–145). Rotterdam, the Netherlands: Sense Publisher.

Leikin, R. (2011). Multiple-solution tasks: From a teacher education course to teacher practice. *ZDM - The International Journal on Mathematics Education*. *43*(6), 993-1006.

Leikin, R. (2014). Challenging Mathematics with multiple solution tasks and mathematical investigations in geometry. In Li Y., Silver E., & Li S. (Eds.), *Transforming Mathematics Instruction*. Advances in Mathematics Education (pp 59-80). Cham, Heidelberg, New York, Dordrecht, London: Springer.

Leikin, R., & Elgrabli, H. (2015). Creativity and expertise: The chicken or the egg? Discovering properties of geometry figures in DGE. In K. Krainer, & N. Vondrova (Eds.), *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (pp. 1024–1031). Prague, Czech Republic: ERME.

Leikin, R., Levav-Waynberg, A., Gurevich, I, & Mednikov, L. (2006). Implementation of multiple solution connecting tasks: Do students' attitudes support teachers' reluctance? *Focus on Learning Problems in Mathematics*, *28,* 1-22.

Leikin, R., & Levav-Waynberg, A. (2007). Exploring mathematics teacher knowledge to explain the gap between theory-based recommendations and school practice in the use of connecting tasks. *Educational Studies in Mathematics, 66*, 349-371.

Levav-Waynberg, A., & Leikin, R. (2009). Multiple solutions to a problem: A tool for assessment of mathematical thinking in geometry. In *The sixth conference of the European Society for Research in Mathematics Education*- CERME-6.

Levav-Waynberg, A., & Leikin, R. (2012a). The role of multiple solution tasks in developing knowledge and creativity in geometry. *Journal of Mathematics Behavior, 31*, 73-90.

Levav-Waynberg, A., & Leikin, R. (2012b). Using multiple solutions tasks for the evaluation of students' problem-solving performance in geometry. *Canadian Journal of Science Mathematics and Technology Education*, *12*(4), 311-333.

Lohman, D. F. (1988). Spatial abilities as traits, processes, and knowledge. In R. J. Sternberg (Ed.), *Advances in the psychology of human intelligence* (*Vol*. 40, pp. 181-248). Hillsdale, NJ: Erlbaum.

Mackworth, N. H. (1965). Visual noise: causes tunnel vision. *Psychonomic Science*, *3*, 67-68.

Mann, E. (2006). Creativity: The essence of mathematics. *Journal for the Education of the Gifted*, *30*(2), 236-260.

Michael, P., Gagatsis, A, Avgerinos, E., & Kuzniak, A. (2011). Middle and High school students' operative apprehension of geometrical figures. *Acta Didactica Universitatis Comenianae – Mathematics*, *11,* 45 –55.

Michael – Chrysanthou, P., & Gagatsis, A. (2013). Geometrical figures in task solving: an obstacle or a heuristic tool? *Acta Didactica Universitatis Comenianae – Mathematics, 13,* 17- 30.

Michael–Chrysanthou, P., & Gagatsis, A. (2015). Ambiguity in the way of looking at a geometrical figure. *Revista Latinoamericana de Investigación en Matemática Educativa – Relime, 17*(4-I), 165-180.

National Council of Teachers of Mathematics. (2000). *Principles and NCTM Standards for school mathematics.* Reston, VA: NCTM.

Nicolaou, S., Gagatsis, A., Panaoura, A., Deliyianni, E., Elia, I., & Televantou, I. (2020). Economic sciences students' understanding on representation tasks concerning the concept of function. In J.-C. Régnier, R. Gras, M. Henry, R. Couturier, & G. Brousseau (Eds.), *Analyse Statistique Implicative. Cadre théorique en relation étroite et au service de multiples disciplines* (pp. 281-302). Besançon: Université Bourgogne Franche-Comté-Besançon: icar.

Palatnik, A., & Dreyfus, T. (2018). Students' reasons for introducing auxiliary lines in proving situations. *The Journal of Mathematical Behavior*, https://doi.org/10.1016/j.jmathb.2018.10.004

Panaoura, G., Gagatsis, A., Lemonides, Ch., (2007). Spatial abilities in relation to performance in geometry tasks. *In Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education (CERME 5)* (pp.1062-1071).

Sawyer, K. (2015). A call to action: The challenges of creative teaching and learning. *Teachers College Record*, *117*(10), 1-34.

Sierpinska, A. (1992). On understanding the notion of function. In E. Dubinsky, & G. Harel (Ed.), *The concept of function: Aspects of epistemology and pedagogy* (pp. 25-58). United States: Mathematical Association of America.

Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM, 3*, 75–80.

Singh, B. (1987). The development of tests to measure mathematical creativity. *International Journal of Mathematical Education in Science and Technology, 18*(2), 181-186.

Stupel, M., & Ben-Chaim, D. (2017): Using multiple solutions to mathematical problems to develop pedagogical and mathematical thinking: A case study in a teacher education program. *Investigations in Mathematics Learning, 9*(2), 86-108.

Torrance, E.P. (1994). *Creativity: Just wanting to know*. Pretoria, South Africa: Benedic books.

Tzefriou, E., Santorinaiou, P., Deliyianni, E. & Elia, I. (2020). Exploring geometrical figure apprehension: the impact of didactic contract. *Mediterranean Journal of Research in Mathematics Education, Vol. 18,* 58-75.

Van den Heuvel-Panhuizen, M. & Buys, K. (Eds.). (2008). *Young children learn measurement and geometry*. Rotterdam, the Netherlands: Sense Publishers.

Yakimanskaya, I. S. (1991). *The development of spatial thinking in school children*. (Soviet Studies in Mathematics Education, *Vol*. 3). Reston, USA: NCTM.

Corresponding Author Contact Information:

Author name: Zoi Geitona

Department: Department of Primary Education

Faculty: School of Education

University, Country: National and Kapodistrian University of Athens, Greece

Email: zgeitona@primedu.uoa.gr

Please Cite: Gagatsis, A., Elia, I., Geitona, Z., Deliyianni, E. & Gridos, P. (2022). How could the Presentation of a Geometrical Task Influence Student Creativity? *Journal of Research in Science, Mathematics and Technology Education,* 5(1), 93-116. DOI: https://doi.org/10.31756/jrsmte.514

Received: 03 August 2021 ▪ Accepted: 06 January 2022