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Didactic proposal to overcome the difficulties in the learning of Area and Volume in Spanish Primary Education students

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Abstract: This work presents a didactic proposal for the learning and measure of surface area and body volume. This proposal is framed in the Anthropological Theory of the Didactic (ATD), based on the recognized errors in the learning of these magnitudes and considering their connection with the typified learning difficulties or epistemological obstacles. The proposal was developed as a didactic sequence, including the tasks from didactic situations (as considered by Brousseau) and with a cross-curricular perspective in relation to the social-systemic structure (ATD), without restricting them in any didactic unit. The praxeology was structured in accordance with the approaches of the ATD, and the didactic methodology was based on the definition of the errors, which followed the phases of development of the usual models in the learning of Geometry. These phases were defined under a generic framework influenced by the developed Van Hiele model for the learning of Geometry. The tasks that composed the didactic sequences were created "ad-hoc" or extracted from adequate sources throughout the Spanish curriculum of Primary Education. The proposal was designed to be applied in the 5th Primary Education grade. The collection of evidences on the students learning regarding the area and the volume after the implementation of the proposal constitutes the natural next step of this project.

Keywords: *Anthropological Theory of the Didactic,Theory of Didactic Situations*; *Errors;Learning difficulties; Area; Volume*; *Primary Education.*

Introduction

This work presents a proposal for a didactic intervention in the frame of the anthropological theory of the didactic (ATD) (Chevallard, 1999; Chevallard, et al., 2015) for the teaching of area and volume in Primary Education (P.E.). The ATD considers the educational institution to be as fundamental as other context aspects in the teaching-learning process. Therefore, in the framework of the ATD, it is not allowed to obviate the institutional restrictions where the educational process is to be developed (Chevallard, 1999). For that reason, in this work the investigation has been developed on the design of a didactic proposal, which could be carried out in the ordinary teaching, in an ordinary center, with its perturbations and ordinary systemic limitations, taking the investigation tools and methods to the institutional and social frame. It has been done by following the ideas of Chevallard (1999), who indicates that institutional borders can be crossed, while remaining inside the systemic structure.

The investigation was focussed in the teaching work, which consisted on the creation of didactic situations as proposed by Brousseau (1986; 1996; 1999; 2000), generated through the connection of different tasks conceived as part of a didactic sequence. The didactic sequence is considered as the element that encompasses the didactic tasks generated in several didactic situations, and, at the same time, this didactic sequence is crossing transversely several levels and didactic units from an explicit selection of didactic situations (Brousseau, 1986).

When the didactic sequence is introduced in the institutional educational frame, a Spanish PE school, it crosses the didactic units, but respects the distribution of the mathematical competences settled by levels (grades) in the above-mentioned institution. This didactic sequence, then, takes part of the didactic units of every level with a certain Praxeology which is directed to overcome the difficulties inherently related to the errors, conceptualized and typified in the literature. This Praxeolgy is also related to other elements of the didactic units of the same level and topic (the Geometry), as well as to the Praxeologies developed for other levels with the same teaching purpose.

The creation of a didactic proposal for the learning of area and volume in P.E., its application and utility, inevitably involves the concepts of magnitude, measurement, area and volume. Nevertheless, since the proposal seeks to be meaningful for the pupil, we must emphasize the meanings of error, obstacle and difficulty, as well as the relation among them. Therefore, the fundamental motivation of the proposal creation is generated from believing that certain types of errors are tied to the construction of the surface area and body volume measurement concepts, and that, at the same time, the conceptual difficulties are, in turn, a consequence of the different types of errors. The innovation of our proposal is the method of confronting the creation of a Didactic Sequence "ad-hoc" addressed to overcoming the difficulties and avoiding the errors, but inside the Spanish education system with its systemic limits, as considered by the ATD framework.

Theoretical framework

The creation of a didactic proposal of empirical-constructivist base which is organized as a prioritized and sequenced set of phases, each phase including didactic activities fitted for the pupil, as well as for the intrinsic difficulty of transforming a knowledge to be taught (defined by the prescriptive curricular frame), in a knowledge necessarily adapted to the pupil, but identifying it as an element of the didactic system, makes us consider these three theoretical models: the Theory of the Didactic Situations (TDS) by Guy Brousseau (Artigue, 2014; Brousseau, 1986; 1989a; October-November 1999; 2000; 2002), the Anthropological Theory of the Didactic (Chevallard, 1997; 1999; Chevallard et al.,1997; Chevallard et al., 2015), and the Model of geometrical reasoning of Van-Hiele and Van-Hiele Geldof (Corberán Salvador et al., 1994; Crowley, 1987; Vojkuvkova, 2012; Van Hiele, 1986; Fouz, 2005; Jaime & Gutiérrez, 1994). In addition to that, we have considered a crucial moment in the teaching-learning process the room implementation of the didactic sequence, so we have also used elements of the Theory of the Didactic Transpositions (Chevallard & Johsua, 1982). Regarding errors and difficulties, we have used the seminal work on the obstacles and learning disabilities of the Mathematics by Guy Brousseau (Brousseau, 1983, 1989a, 1989b). In summary, the present study seeks to be a bridge connecting different theoretical frames which are not exclusive among them, in the sense defined by Trigueros (November 2016), aligned with works of other authors which, on the contrary, exhibited a different perspective (faltaría alguna referencia).

In the Anthropology of the knowledge in Mathematics Education it is raised that the stablishment of any teaching and learning project is obtained under the identification of the knowledge: Scholarly Knowledge as the theroretical and mathematical knowledge; Knowledge to be taught, as the content to teach; Taught knowledge, as the knowledge which has really been taughtto the classroom ; and the Learnt Knowledge, as the object learned by the pupil. The process of transformation from the Scholarly knowledge to the learnt knowledge, is called by Chevallard as Didactic Transposition (Chevallard et al , 1997; Chevallard, 1997).

According to Chevallard, the mathematical activity, like any another human activity, is characterized by a praxis, praxeological organization or praxeology (Chevallard, 1999), which is constituted by a practical-technical block and a theoretical-technological block. Both are the main components of the praxeology. The practical-technical block is composed by the tasks, T, which can be proposed in several levels, and the set of ways to achieve them, or the techniques, θ. At the same time, the set of tasks is composed by different individual tasks, t. Therefore, for this author t ϵ T. For example, a type of tasks is *to calculate* (T) and a concrete task *to calculate areas of flat figures* (t). Meanwhile, the theoretical-technological block consists of a technology, Φ, and a theory, Θ, which is used to justify the technology. At the same time, the specific way of facing each task (t), θ, is known as a technique, belonging to the technology, Φ. That is, a technique, θ, belongs to a technology Φ, provided that θ ϵ Φ. Therefore, the four elements [T, t, θ, Θ] are connected in such a way that a praxeology relative to a type of tasks contains several individual tasks, which are the technology. Therefore, according to Chevallard, the practical technical block, [T/t/θ/Θ], is constituted by a certain type of tasks, T, and a certain task, t, that is solved by using a specific technique, θ, inside a theory, Θ, which constitutes the final knowledge on how to do, and the knowledge itself. The entire set [T/t/θ/Θ] constitutes a particular praxeology, where *particular* means that it is a praxeology addressing only a type of tasks, T.

The didactic aspect in the teaching of Mathematics is created to avoid errors and obstacles, whose nature is not only epistemological. For the error analysis, we considered Brousseau (1980, 1981, 1986, 1996, 2007) categorization. This author suggested that the construction of the above mentioned situations, the didactic situations, is a result of investigation. Therefore, these didactic situations are neither universal nor designed in a standard way. On the contrary, the didactic situations depend on the mathematical concept, the way and the institution they are going to be carried out. To construct them in a suitable way, Brousseau proposed to consider several aspects: the meanings of the concept inside the structure of this theory; the historical and cultural conditions in which the concept emerges; the intermediate ways of appearing, the conceptions and perspectives that became obstacles with regard to the evolution of the concept; the problems that lead, or have driven, to the overcoming of these obstacles, and that have allowed a later development; the study of the psychogenesis of the concept, or its genetic epistemology; a didactic analysis of the pretended meanings of the concept, and/or those transmitted by its teaching, both in the present and in the past; and, eventually, the study of the didactic transposition.

In this work we propose a specific didactic transposition involving a praxeology. That praxeology consisted of a didactic sequence composed of tasks, created either “ad-hoc", or gathered from the literature, always with the target to overcome the errors identified in the literature regarding the concepts of *measurement of surface area and solid volume*.

The whole theoretical framework used in this work is summarized in Figure 1. It includes the “Knowledge” and the “to know how”, inside the praxelogy, containing also the construction of tasks, techniques, technologies and theories required for the construction of the proposed didactic sequence:

To clarify what is understood in this work by didactic sequence, figure 2 is prepared, by considering the context of the ATD (Chevallard, 1999), the didactic situations (TDS) according to Brousseau (1983), the didactic unit as part of the cultural environment of an educational system (institutional) where this curricular development holds, which is Spain.

Figure 3 shows the conceptual map that starts from the praxeology created, and that develops the tasks as didactic situations (TDS), creating “ad-hoc" chains of tasks compatible with the phase structure. This sequence is influenced by the development of the different stages and levels of the Spanish official curriculum of Geometry for P.E.

Errors, Difficulties and Misconceptions

The first conceptualization of the errors in Mathematical Education was presented by Brousseau (Brousseau, 1976, 1980, 1981, 1983, 1989a), who thought that these could be of three types: ontogenetic, epistemological and didactical errors. Other authors as Roan et al. (2008) changed the term ontogenetic by cognitive, that was later changed to psychological.

In the German and Anglo-Saxon context, it is a classic work the error review of Radatz (1980), which exhibits the different theoretical approaches. In this work, the errors were detected in the learning of the students, and were qualified as systematic, persistent and prolonged, if the teacher does not intervene. Radatz analyzed also the causes, and considered the errors as derived from cognitive difficulties, from the communication process and interpretation of information, or from the interaction of variables that take part of the didactics of Mathematics.

Regarding the causes of the errors, Kuzmitskaya and Menchinskaya (1997, cited by Engler et al., 2004; Engler et al., 2006), in their study on the difficulties found in the process of solving mathematical problems by P.E. children with a light mental deficiency, identified as possible causes the insufficient short-term memory, the insufficient comprehension of the problem conditions, the lack of handling the calculation rules, and the incorrect use of the four basic operations.

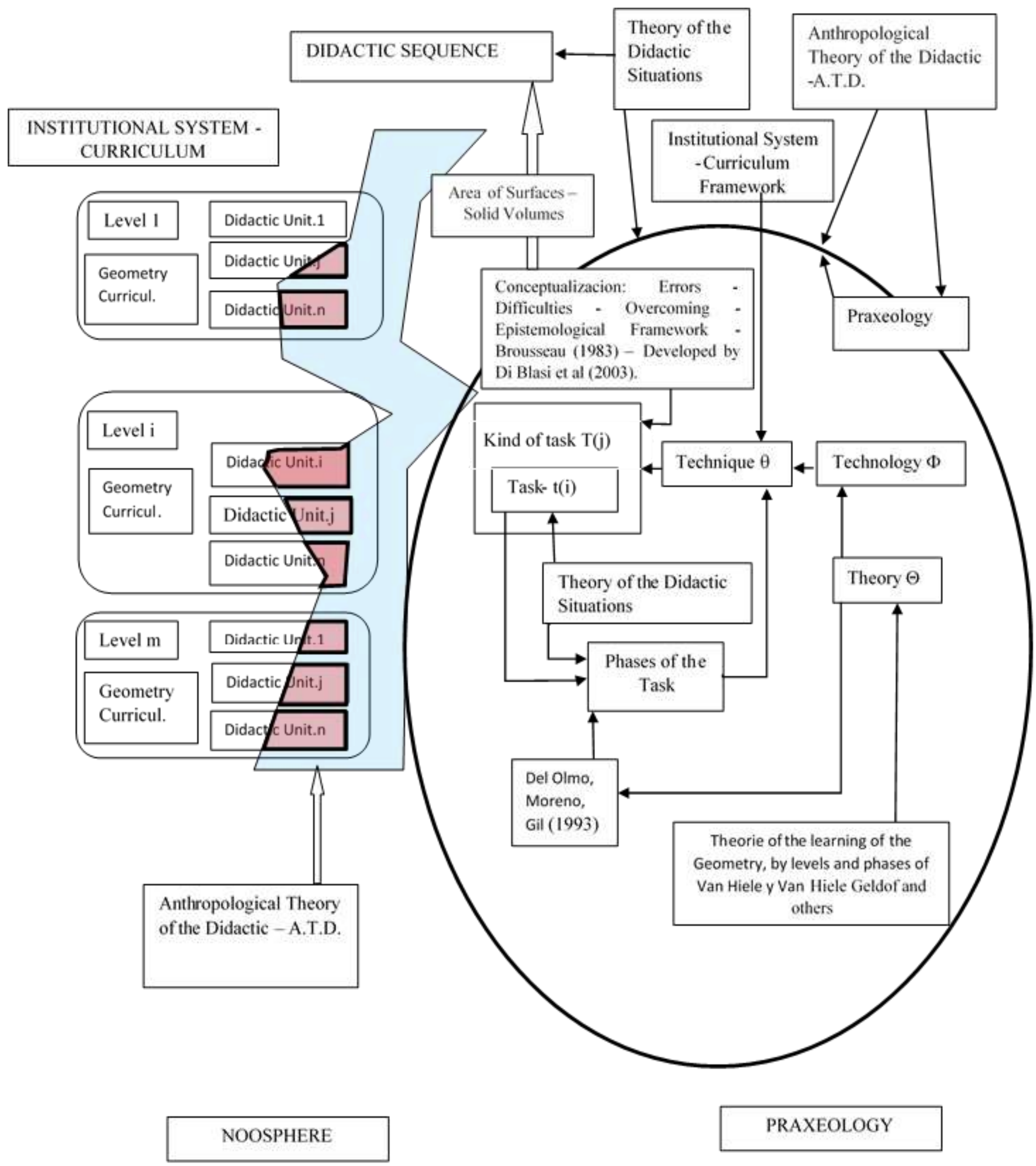

Figure 1. Description of the construction of the Didactic Sequence through a praxeology, taking into account the accepted systemic structure (levels on the left), the curricular transversality of the didactic sequence, the elements of the ATD (noosphere), the TDS, the context of the medium, and the leaning objectives, which determine the techniques, technologies, and the phases followed to reduce errors and overcome difficulties or obstacles (Source: Authors from ATD, TDS, Van-Hiele and the reference included in the figure).

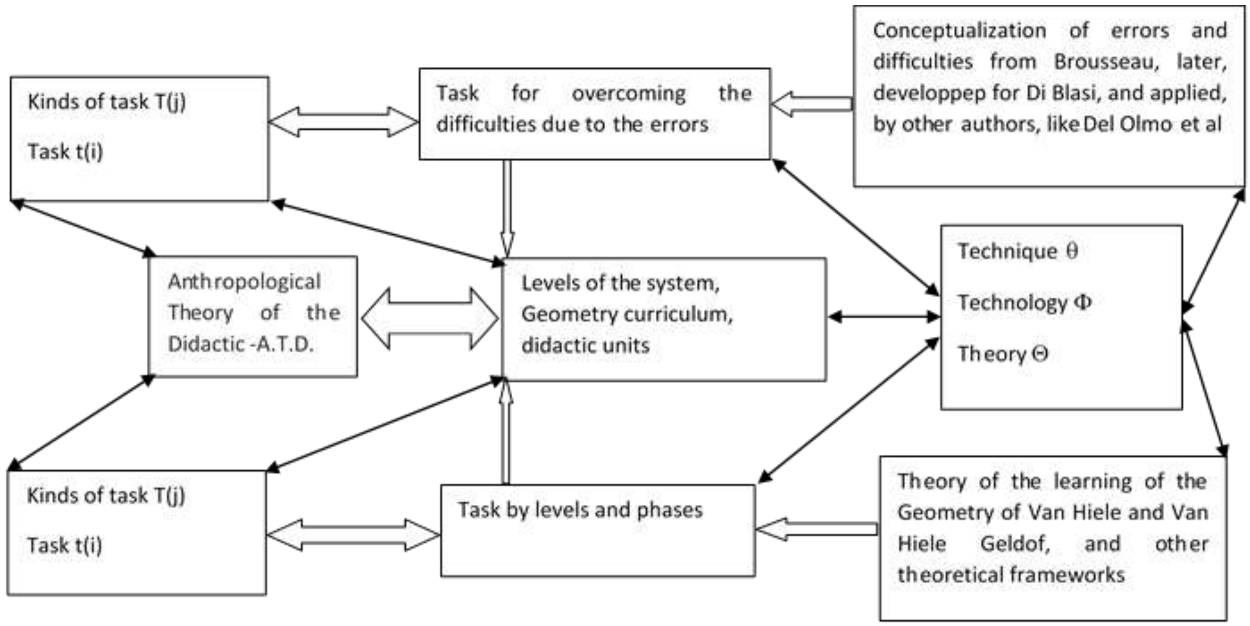

Figure 2. Genesis of a praxeology (Chevallard, 1999) for the construction of tasks from kinds of tasks, aimed at overcoming difficulties that produce errors. The tasks are informed by the conceptualization of errors and difficulties in learning, respecting the systemic structure established in the Noosphere of the teaching-learning process of Mathematics, using methods developed in phases, settled in frames like the theory of The Van Hiele. (Source: Authors from the indicated references)

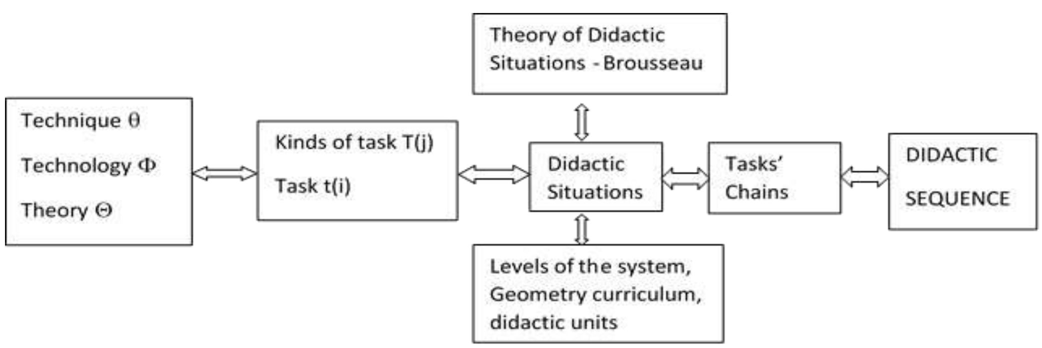

Figure 3. Conceptual map that connects the Praxeology with the theoretical frameworks, and with the construction of the Didactic Sequence as a chain of tasks framed in didactic situations (Source: Authors from ATD, TDS, Van-Hiele)

In Heinze's work (2005) the author asked about what can be done with errors that are going to take place, or that take place, and about the ways to turn errors into a tool for the design and transformation of the learning, that is, the ways to make them useful for the learning of Mathematics. Although the answer to these questions would be fundamental to foster the advance in Mathematical Education, the general line in the works on errors is the creation of different compilations merely descriptive, like the work of Özerem (2012), or focused on an operational categorization in the different areas of Mathematics

(Nicky, 2014), or proposing ad-hoc error classifications based on the concept the error is related to (Cabello Pardos et al., 2014).

To make errors fruitful for the learning of Mathematics, aligned with the answer to the questions raised by Heinze (2005), only the classification of the errors proposed by Di Blasi Regner et al. (2003) is found, where errors are related to learning difficulties due to misconception. The author considered 5 types of errors, according to the agent that causes them, such as the difficulties associated with: a) the complexity of the mathematical objects, b) the mathematical thinking processes c) the teaching processes, d) the pupils' cognitive development, and e) the affective and emotional attitudes.

Bearing in mind the paragraphs above, in the present work the epistemological meaning of significant errors of Bachellard (2000; 2004) was taken into account, considering them as epistemological obstacles. In addition to that, from a practical point of view, the error idea of Rico (1995), who considered them as a result of a deficient or incomplete knowledge of the pupil, and as constituent elements of the teaching-learning process, was followed. Moreover, for the classification of the errors, the seminal works by Brousseau (1976, 1980, 1981, 1983, 1989a) are considered. Taking that into consideration, in table 1 is shown the relation that the present work authors have established between errors (Brousseau, 1983) and the learning difficulties according to Di Blasi Regner et al. (2003).

Table 1

Relationship between errors and learning difficulties associated to different sources of misconceptions

(Source: Adapted by authors from the indicated references)

Guy Brouseau (1983)	Di Blasi Regner et al. (2003)
1) Ontogenetic (cognitive or psychological)	d) Difficulties associated with the cognitive development of students. e) Difficulties associated with affective and emotional attitudes.
2) Epistemological	a) Difficulties associated with the complexity of mathematical objects. b) Difficulties associated with mathematical thinking processes.
3) Didactic	c) Difficulties associated with teaching processes.

Conceptualization of Curricular Elements: Magnitude, Area and Volume

Any research process must be developed in coordination with the curricular framework established for the didactic problem, with the ages and with the cognitive levels of the students. In this way, it is much more likely to be reproduced or compared with others. As stated in the work of Díez et al. (2016), where the authors make a comparison of Spanish curricula from 1945 to 2013, the methodological orientations for Primary Education have followed a similar didactic and curricular scheme, in spite of the fact that, in this time period, four educational laws have existed in Spain. However, in spite of the legislation changes, the concepts of magnitude, area and volume are considered as curricular contents, and their introduction at similar ages. On the other hand, it should be noted that the last two laws (Organic Law 2/2006 and Organic Law 8/2013) formulated a framework that allows for the development of didactic situations such as those included in the didactic proposal presented in this work.

There are concepts of magnitude of a physical or technical type, appropriate to other fields, but not to that of didactics. However, the didactic concept of magnitude collected by Godino (2004) "...is any aspect of things that can be expressed quantitatively, such as length, weight, speed or luminosity" (p.295), is the one considered in this work. Therefore, area and volume are considered derived and geometric magnitudes in our educational context and work, unlike length, which is a fundamental magnitude. Area and volume are also extensive magnitudes, in the sense that the magnitude of a body is the sum of the magnitudes of the summand bodies.

The area and volume magnitudes are crucial for the development of knowledge on the environment. To quantitatively express a magnitude, it is measured. And measuring is a process that involves several concepts (Chamorro & Belmonte, 1988). When we measure, we associate a number with a quantity of magnitude. The number is the result of comparing the quantity of magnitude with a reference that we call a unit. In addition to that, the importance that the institution assigns to these magnitudes, as Godino et al. (2002) indicated, is evidenced through their presence in the Mathematics curricula from Infant to Secondary Education.

To address the measurement of areas we assume that, following Lovell (1986), area is the magnitude that measures the amount or extent of surface a body has. The area as magnitude is not linked to the shape of the body, but the surface is a manifestation or property dependent on the shape of the body. The area, therefore, is not linked to the shape of the surface that it quantifies, although in many cases it is linked to the measurement procedure, or to the process followed for its calculation. In order to develop the notion of area, flat figures and regular bodies are conceptually very accessible objects for Primary Education students. That is why we focus on them in this work.

For the didactics of Mathematics, the introduction of the difference between surface and area is a controversial topic. We find authors such as Tierney, Boyd and Davis (1990, cited by D'Amore & Fandiño Pinilla, 2007), where the area is directly related to the algebraic arithmetic tool directed to its calculation, omitting the importance of the relational conceptual element area-surface. On the other hand, the work of Piaget, Inhelder and Szeminska (1960) defends their importance and the need to approach them with suitable didactic instruments. In this work it is also reported that children from the age of seven perceive operatively the conservation of the area when the figures are altered, for example, changing their position; but until the age of eight or nine they do not fully understand the application of a unit of measurement. In addition, they propose empirical processes divided into phases to bring children closer to the acquisition of these concepts.

These distinctions require the concept of surface. In the work of Chamorro (1997, cited by D'Amore & Fandiño Pinilla, 2007) the author emphasizes the difficulties that arise from the concept of surface and its analysis, highlighting the importance of the differentiated introduction of the surface with respect to the area.

The other magnitude of interest in this work is volume. Volume is the magnitude that measures the amount of space a body occupies and encloses the notion of three-dimensionality. This concept is usually confused with the concept of capacity, since, although they are different magnitudes, they are also intrinsically related, since the capacity of a container to house another body coincides with the volume of the inner space delimited by the surfaces of the container. For Del Olmo et al. (1993) volume and capacity are different in terms of attributes, since volume is usually understood as occupied space and capacity as empty space with the possibility of being filled. In many aspects of the present study, the work mentioned above is considered as a reference that orients well the construction of a praxeology.

From a didactic point of view, we do not intend to break the accepted systemic social framework, and, in this sense, we find inappropriate at a Primary Education level purely procedural approaches based on definitions of metrological dictionaries, precisely because we seek the construction by students of clear and meaningful operational definitions for them. For example, a definition of the type of magnitude such as "attribute of a phenomenon, body or substance that is capable of being qualitatively differentiated and quantitatively determined", seems to us to be inappropriate in a P.E. framework. The student at this stage must first approach all the concepts implicit in this type of decision in order to understand them, to structure them and to assimilate them without error, and, at the same time, to avoid making it difficult for them to learn in the future. On the other hand, from the point of view of Vygotsky's social constructivism, we need to construct the concepts in the minds of the students from their zones of proximal development towards the advances that we intend to promote. In this way, it will sometimes be easier to address the connections of body volume to surface area, and from this to line length, and the opposite way around, from line length to surface area, and from this to body volume, than to do so without connections, for example, by means of the isolated presentation of length, area or volume. We assume that it is feasible that area and volume can be qualitatively differentiated and quantitatively determined by methods that do not depend on length. But at the same time, we will assume that it is feasible, and that it has been made since immemorial time, the opposite process of ascending line-length, to surface-area, and from this to body-volume, and that it is didactically carried out.

The Scholarly knowledge is represented in Spain by the curriculum, which includes the area and volume magnitudes as topic. Table 2 includes a summary of the Spanish curricular framework for area and volume in P.E.

Table 2

Mathematics' Curriculum divided into blocks. These are specifically in 3rd and 4th grade, and in 5th and 6th grade of Primary Education. (Source: Authors from law)

	3rd and 4th grade	5th and 6th grade
Block 1: The understanding, representation and use of numbers: operations and measurement.		- The measure: estimation and calculation of surfaces. - Election of the unit and the most suitable instruments to measure and express a measure. - Utilization of surface units. - Comparison of surfaces of flat figures.
Block 2: Interpretation and representation of shapes and the situation in space.	- Identification and classification of plane and spatial figures in everyday life. - Construction of flat figures from a development. - Exploration of elementary geometric shapes. - Comparison and classification of figures and geometric bodies using different criteria.	- Formation of flat figures and geometric bodies from others by composition and decomposition.

Procedural Elements: Connection Between Errors, Difficulties due to Misconceptions, and Phases for the Teaching-Learning of Area and Volume

As indicated in the previous section, a few authors provide didactic guidelines to try to correct the errors detected in the students, which would contribute to the improvement of the process of teaching-learning Mathematics. Among these works is that of Del Olmo et al. (1993), who emphasize the complex aspect of the measurement of areas beyond the mere use of simple formulas; the underlying complexity to be measured; and the conceptualization of the measurement of areas. They consider that the student who has not well reached the concept of the area does not usually understand the algorithmic use of formulas because they lack meaning for him. For this reason, these authors state that the formula is the shortest way to reach the result, provided that the spontaneous and proper methods of each student have been previously developed. These authors consider that the structures for teaching and learning with respect to didactic approaches in relation to area and volume measurements would be easily generalizable for other geometric magnitudes such as length, for example, and they highlight different phases in the didactic process of assimilation of the construction of the magnitude measurement:

1st Phase: Perception and Comparison of the quality to be measured. Relational comparison of the quality with order relation structures.

2nd Phase: Measurement with non-conventional, manipulative or constructive units.

3rd Phase: Measurement with International System units, valuing the conventional aspects, approaching them to the modelling, for different systems of measurement.

4th Phase: Arithmetization by using sequences of activities and tasks until the most complex verbal statements are reached.

5th Phase: Estimation, assuming and promoting its need.

In their didactic orientations for the perception phase, they propose different didactic strategies, such as drawing and colouring body traces, observing fruit peelings or covering objects with paper. They also propose the use of structured materials such as tangram, polyminoes and polydiamonds. For comparison, they propose the comparison of areas of flat figures by cutting and overlapping them. For the phase 2, which involves using a non-standard unit of measurement, they propose as an example the use of polygons that cover the plane perfectly, by means of graphics and visual aids to show the relationship between the different units of the international system (I.S.).

In terms of volume, Freudenthal (1983) states that this concept is less exposed to phenomenological impoverishment than the area, due to the double aspect of capacity and volume. For the construction of the mental object of volume in children, Freudenthal proposes the following sequence of phases: the first, to begin with transformations of breaking and remaking bodies with constructions; the second, to work the equivalence of capacity of containers and volume of solid bodies; the third, to use real transformations of emptying to compare content; the fourth, to approach the transformations that conserve and do not conserve volume. This last phase is not proposed for a specific age of the learner, although some references to conservation and age are collected in the work of Piaget, Inhelder and Szeminska (1960), who conclude that students do not acquire the notion of conservation until the end of the stage of formal operations, when they are 11 or 12 years old.

Regarding the didactic proposals for the volume, Lunzer (1960, cited by Del Olmo et al., 1993) reported that, in a test with children aged 10 to 14, none understood volume as being surrounded by limiting faces. Unlike Piaget et al., (1960), this author stated that the conservation of the volume arose between the ages of six and eight, but, at the same time, he referred to the lack of activities for understanding the concept in teaching practice as the cause for delaying its acquisition. Lovell and Ogilvie (1961, cited by Del Olmo et al., 1993) studied the notions of internal volume and dislodged volume through experiences with students. They asserted that it is possible for children to learn more quickly about volume if appropriate experiences were carried out at school.

Therefore, gathering the ideas of Freudenthal (1983), it can be considered that, for the understanding of the magnitude of volume, the didactic orientations recommended by Del Olmo et al. (1993) would be adequate for the area: integral study of quality and its measurement; comparison of objects with respect to magnitude; raising the need to have a unit of measurement to quantify volumes with respect to it, and introducing the different units of volume applying it to daily life problematic situations. In this line, we worked on the proposal presented in this work for the study of areas of surfaces and volumes of bodies. For its design, the connections established between the observed error and the learning difficulty in the concepts of area and volume, detailed in the Figure 4, have been elaborated.

On the other hand, table 3 shows the connection between the errors selected from different approaches and their susceptible cause or explanation, based on Del Olmo et al. (1989).

In table 4 we can observe the main selected errors and their incidence in the area and volume measurement processes.

Didactic Proposal

This section presents a didactic proposal based on the concept of didactic sequence according to Zabala (1998), on a transversal to generic structure according to Tobón et al. (2010), and on the concept of didactic situations in the sense given by Brousseau (1981, 1986, 2007) and Chevallard (1987). The methodological phases proposed by Del Olmo et al. (1989), Camacho Martín et al. (2003), and Godino (2004) and Godino, Batanero and Roa (2002), following epistemologically the models of V.H. (1986), Corberan (1994), Crowley (1987), Fouz, (2005), Vojkuvkova (2012) and Brousseau (1986; 1996) have been taken into account for the linking of the didactic sequences and the development of the proposal. The phases in the didactic proposal have the objective of logically sequencing the way in which primary school students acquire the concepts of area and volume, and of avoiding or, at least minimizing, the recognized errors related to the conceptualization of these two magnitudes.

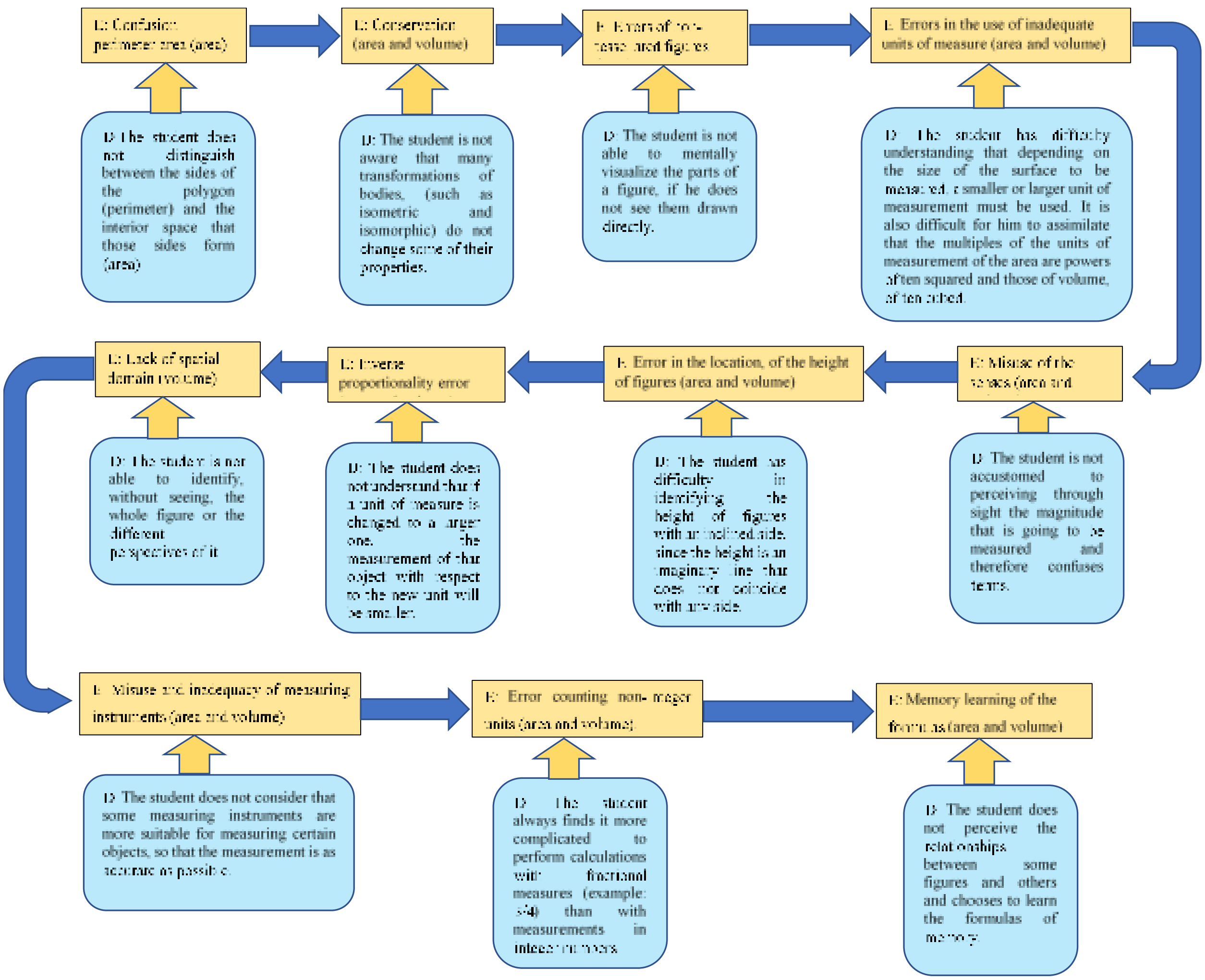

Figure 4. Connections between observed error and learning difficulties (or source of misconception) on areas and volumes of figures in Primary Education. E: Errors, D: Difficulty (Source: Authors from references)

Table 3

Table of errors with their explanation and examples

Error	Explanation	Example
Confusion perimeter-area.	Wagman (1982, cited by Del Olmo et al., 1993), found that one third of the subjects who intervened in the study confused the area with the perimeter. (It may be due to the methodology used).	In a drawing, they are asked to determine the area of a square of 2 cm on each side. The student answers that the area is equal to 8.

Conservation of area and volume.	The conservation of the volume or area is that they do not change, with some transformations. The volume is a magnitude, whose conservation costs the students, Piaget, Lovell, Ogilvie, and Freudemthal.	If we cut a leaf, into pieces, its area remains the same before and after the transformation. Apply a deformation to a plasticine ball forming a sausage.
Measurements errors.	Hart addresses the following difficulties: - That the figures are more complicated than the rectangle. - That the figures do not appear paved. - The inverse proportionality between the size of the unit of measurement and the figure. - Counting non-whole units.	If the figure is not paved (not tessellated), students apply formulas as the only strategy. If the unit of measure is changed to a larger one, for students the measure is greater, being smaller (inverse proportionality) With fractional units the error is more likely.
Errors attributed to the traditional methodology (areas and volumes)		
1. Misuse of the senses.	Del Olmo et al. (1989), states that the first step in the process of measuring a magnitude begins with the perception of the quality that is going to be measured, from the infancy of the individual.	Students do not differentiate between the area and the volume of an object.
2. Use of inadequate instruments	Using, alone, conventional measuring instruments, makes the choice a little lucky.	Use the ruler to measure the length of a curve.
3. Abuse of accuracy in measurements	It is often confused, as an integer measure, with exact measurement, the integer being understood as exact.	Students say, 6.5 cm^2, it cannot be an exact measurement.

4. Lack of strategies to measure common objects	Students feel it easier to solve area problems on a regular basis. When they must calculate an irregular surface area, the student does not know how.	It is necessary to calculate the area of a starry shape.
5.Lack of spatial mastery	Little ability to mentally use a figure. It is hard to visualize parts of a body with volume represented on a piece of paper.	The student does not know how to draw different perspectives of the same figure.

Table 4

Errors and measures of area and volume: relationship among them in P.E. (Source: Authors from references)

Errors	Area	Volume
Confusion perimeter-area memory learning of the formulas.	✓	
Conservation.	✓	✓
Measurement errors.	✓	✓
Errors of unpaved figures.	✓	
Errors in the use of inadequate units of measure.	✓	✓
Lack of spatial mastery.		✓
Inverse proportionality error.	✓	✓
Location height error of figures.	✓	✓
Misuse of the senses.	✓	✓
Misuse and inadequacy of measuring instruments.	✓	✓
Error counting non-integer units.	✓	✓
Memory learning of the formulas.	✓	✓

Figure 1 summarizes the considerations and connections for the theoretical frameworks assumed for this work. Within the ATD, the tasks will be assumed to be each activity, which are assimilated to a technique, according to the phases in which the kind of task is classified. It is shown in table 5.

Table 5

Correspondence between techniques and phases of the task. (Source: Authors)

Technique (Activity, ɵ)	Task’s Phase (T)
Use real materials that are close to the students' environment.	Task included in the Magnitude Perception Phase.
Perform transformations, conservation and activities with meshes.	Task included in the Comparison Phase.
Approach the measure with paving figures in the case of the area, and tessellation in the volume, and then introduce the international system of units with multiples and submultiples of m^2 and m^3.	Task included in the Measurement Phase.
Use materials such as grids to introduce the length and width of figures, and the depth in the case of volume. Later the technique must relate some activities with others in order not to have to memorize an excess of formulas.	Task included in the Arithmetization Phase.
Move from the measure to the estimate using the arithmetization.	Task included in the estimation Phase.

The didactic sequences were designed for 4^{th}, 5^{th} and 6^{th} grades and were distributed according to methodologically and didactically chained phases in such a way that some of them overlapped. The aim of this overlapping was to ensure progression in student learning. The organization by levels (grades) is influenced by the Spanish curricular framework and constitutes a systemic and epistemological limitation recognized in ATD, also considered in this theory as inevitable and determinant of the educational didactic process. The scheme can be seen in Figure 5.

4^{th} grade→ Area: 1^{st} Phase (perception phase) + 2^{nd} Phase (comparison phase) + 3^{rd} Phase (measurement phase).

5^{th} grade→ Area: Previous phases + 4^{th}Phase (arithmetization phase) + 5^{th} Phase (estimation phase). Volume: 1^{st} Phase (perception phase) + 2^{nd} Phase (comparison phase) + 3^{rd} Phase (measurement phase).

6^{th} grade→ Volume: Previous phases + 4^{th}Phase (arithmetization phase) + 5^{th} Phase (estimation phase).

Figure 5. Phases used in the didactic sequence for 4^{th}, 5^{th} and 6^{th} grades of P. E. (Source: Authors)

The application of the proposal for purposes of case study and research, in pre-experimental design, was carried out in the 5th P.E. grade, making a synthesis of what was designed for the 4th, 5th and 6th of P.E., in accordance with the curricular design of the school, and also suitable for experimentation. Therefore, the design of the proposal reached in its transversality the three courses, that is, all the curricular extension in which these geometric magnitudes appear in P.E.. However, its application was carried out for a single course. To this end, it was compacted in some aspects in order to be able to evaluate more easily its future applicability.

The established relationship between the errors and the five methodological phases selected for the teaching of the area and the volume is shown in table 6:

Table 6

Errors overcome in each methodological phase (Source: Authors)

	Area	Volume
1st Phase	- Misuse of the senses. - Perimeter-area error.	- Misuse of the senses. - Volume conservation errors.
2nd Phase	- Area conservation errors.	- Errors of lack of spatial mastery.
3rd Phase	- Errors of unpaved figures. - Use of inadequate units of measurement.	- Proportionality error. - Use of inadequate units of measurement.
4th Phase	- Use of inadequate units of measurement. - Error of location of the height of figures. - Recording learning formulas.	- Error of location of the height of figures. - Recording learning formulas.
5th Phase	- Use of inadequate units of measurement.	- Use of inadequate units of measurement. - Misuse of the senses.

The proposal lasted for 10 sessions of different durations, during which the phases were alternated. The activities designed for each task, the kind of task, the phase in which it was applied, and the objective pursued are included in Figures 6a and 6b.

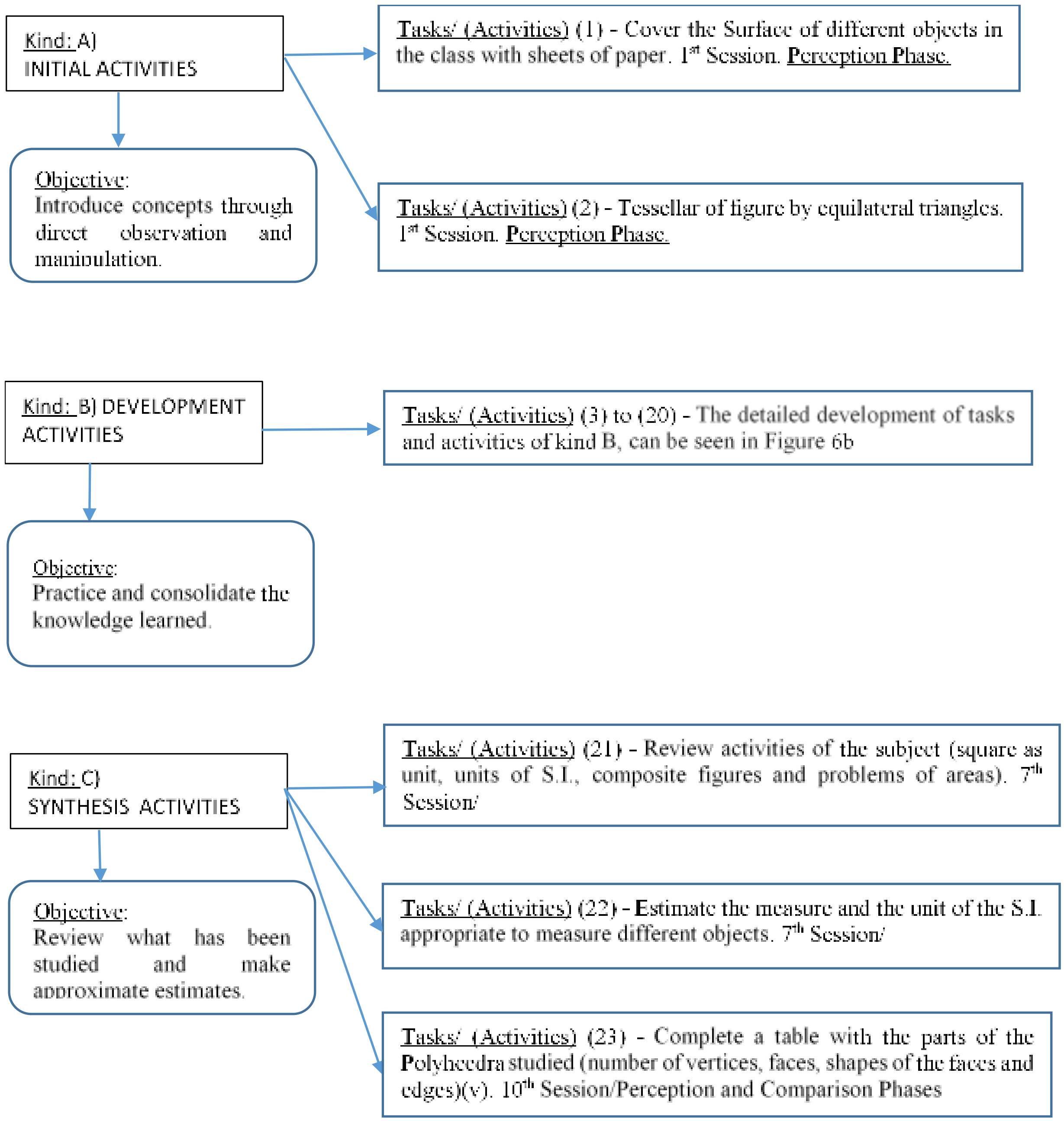

Figure 6a. Activities or tasks, kind of tasks, phase, session and objective they were addressed. Detail and general organization for the different kind of activities (v:volume)

Kind: B) DEVELOPMENT ACTIVITIES

Objective:
Practice and consolidate the knowledge learned.

Tasks/ (Activities) (3) - Write the area of each figure counting the squares as a unit. 1st Session. Perception Phase.

Tasks/ (Activities) (4) - Observe two figures and compare their areas and their areas and theirs perimeters. 1st Session. Perception Phase.

Tasks/ (Activities) (5) - Count units of squares to find your area. 2st Session. Perception Phase.

Tasks/ (Activities) (6) - Construct squares or rectangles of different sizes with pentomines and then indicate the area or perimeter of the resulting figures. 2st Session.

Tasks/ (Activities) (7) - Surround the figures that have the same area despite the difference of their forms. 2st Session.

Tasks/ (Activities) (8) - Make figures and cover silhouettes using tangram pieces. 3st Session. Measurement Phase

Tasks/ (Activities) (9) - Make figures with cross-linked paper and relate the width and length in a table to calculate the area. 3st Session.

Tasks/ (Activities) (10) - Observe and reason the relationships between basic plane figures, to know where the formula comes from to calculate the area. 4st Session. Arithmetization Phase

Tasks/ (Activities) (11) - Measure and calculate the area of squeares and rectangles. 4st Session.

Tasks/ (Activities) (12) - Find the area of triangles from the area of squeares or rectagles. 4st Session.

Tasks/ (Activities) (13) - Solve problems of daily life (area of a cardboard, area of a cork, area of a plot) calculating the corresponding area. 4st Session.

Tasks/ (Activities) (14) - Carry out a human scale, representing the units of measure of area. . 5st Session. Arithmetization Phase

Tasks/ (Activities) (15) - Convert some units into others. . 5st Session.

Tasks/ (Activities) (16) - Observe and calculate the area of composite figures. 6st Session.

Tasks/ (Activities) (17) - Observe and calculate the area of composite figures. 6st Session.

Tasks/ (Activities) (18) - Directly observe the relationship of experiences, think hypotheses, and answer related questions don conservation of volume (v). 8th Session/Perception Phase

Tasks/ (Activities) (19) - Spatial visualization activities (v). 9th Session/Perception and Comparison Phases

Tasks/ (Activities) (20) - Construct five regular polyhedrons and round bodies with paper (v). 10th Session/Perception and Comparison Phases

Figure 6b. Activities or task, phase, session and objective for the Development Activities (kind B). Note: the activities on volume are referred as (v); the rest refers to area. (Source: Authors).

As an example, two adaptations of the specific tasks included in the didactic sequences for 4th P.E. grade are included in table 7.

Table 7

Arithmetization for 4th of Primary Education (Source: Authors)

4th grade of Primary Education	
4thPhase (Arithmetization Phase) Task 1. (Area)	
Specific objective of the task	Show the relationship between the areas of some figures and others so that they do not have to memorize the formulas, emphasizing the location of the height and base of the figures.
Errors to avoid	- Memory formulas learning. - Height location error of the figures.
Intrinsic difficulty	Inability to visualize the relationships between basic plane figures.
Task: Students visualize the relationships between rectangles, triangles and rhomboids, attending to the decomposition of polygons in figures, and obtaining formulas.	

In the ATD, connected to the "taught knowledge" is the "knowledge to be taught" and, as a pure mathematical support, the "wise knowledge". The didactic transposition was oriented towards the objective of enabling the passage from the manipulative phase, based on direct sensory perception (figure 7), to the concepts abstraction, such as the visualization of relationships among the basic flat figures (table 7). Among the designed constructive activities, some are shown in figure 5.

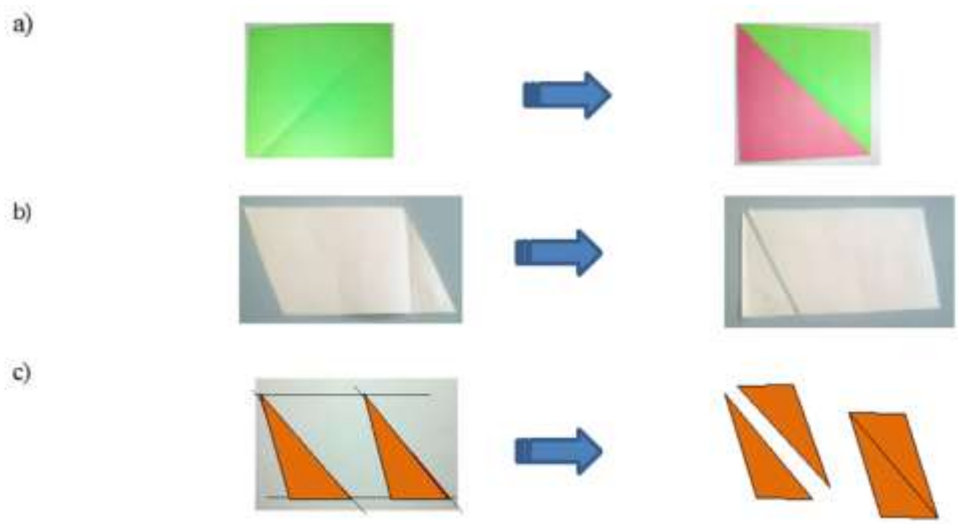

Figure 7. a) Relationship between a square and an isosceles right triangle: a square is formed by two isosceles right triangles, so the area of the right isosceles triangle is half that of a square whose side measures the same as the equal sides in the triangle. b) A rhomboid (quadrilateral) can generate a rectangle parallelogram if one of the two triangles (see folded part) is transferred to the other end of the figure. c) Two identical obtuse angle triangles can form a rhomboid parallelogram after the appropriate translations and turning. (Source: Authors)

In order to decouple the "wise knowledge" and the "taught knowledge" from the manipulative material used (Font, 2003), activities involving the use of different tools were designed (See Figure 7 and link).

Table 8

Task designed for the study of volumes including the definition, error and assigned difficulty (Source: Authors).

5th grade of Primary Education	
Phase 1st and 2nd. (Perception and comparison Phases) - - Task 1. (volume)	
Specific objective of the task.	That the students have a first contact with the concept of volume through the realization of direct experiences and visual stimulation.
Errors to avoid.	- Misuse of the senses. - Volume conservation.
Intrinsic difficulty.	The student is not accustomed to perceiving through sight the magnitude that is going to be measured and that is why terms are confused.

	In the conservation of the volume the student does not understand that in spite of the fact that the object suffers a transformation of its initial state, it continues occupying the same space and therefore has the same volume.

Task: Reflection and visualization of volume conservation. Difference between internal volume and external volume.

The following figures show activities, tasks and experiments carried out by the students as part of the study of volumes.

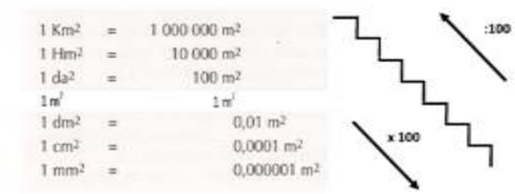

Figure 8. Surface I.S. units'relationships (Source: Rieiro & Martínez, The Measure and the Magnitude, Ed.Granada, 1993).

Figure 9. Students performing the I.S.units' activity in the school staircase. (Source: Authors)

Figure 10. Containers used to perform the volume conservation experiment (left). Transformation caused when molding / deforming a piece of plasticine (right). (Source: Authors)

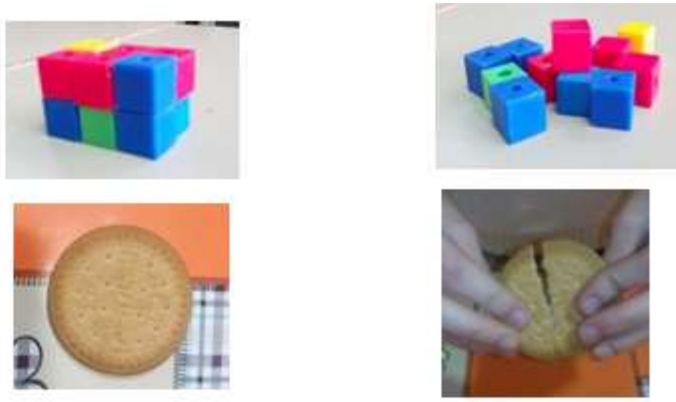

Figure 11. Transformations that conserve volume: ordered and disordered building blocks (top left-top right), whole and broken cookie (bottom left-right). (Source: Authors)

To see the complete didactic proposal, click on the following link:

https://pruebasaluuclm-my.sharepoint.com/:b:/g/personal/ignacio_rieiro_uclm_es/EX4ZpHTygPFMng3w4XaBjQgBZBWsDUGx0h1PjHhmvX4BXw?e=2zB3si

To see the dicactic proposal carried out in 5th P.E. grade, click on the following link:

https://pruebasaluuclm-my.sharepoint.com/:b:/g/personal/ignacio_rieiro_uclm_es/EZPYqQ6Wm6NJn3wfpnU1bSUBAweNAIQ2qOgt53XRhl0C5w?e=xQEN0o

Conclusions and Prospective

In this work, a didactic proposal connecting three different theoretical framework has been created. It has been based on the errors identified in the literature in relation to the construction and measurement of surface area and body volume, within the framework of the ATD, including didactic situations linked by didactic sequences within the framework of the Theory of didactic situations by Brousseau. In line with this last author, it has been proved that the teacher's work either starting from existing activities or from others elaborated on their own, has an important investigative base, which could be completed with the verification of the effectiveness of the proposal through its use in the classroom, and an ulterior semiotic error analysis. As a result, a procedural structure has been achieved, preserving the systemic structure, but adapting it to the needs of the students through the incorporation of the curriculum, taking into account the appropriate sequence in the phases of the process, implementing an operative and flexible praxeological structure, and also foreseeing the obstacles, the errors and the difficulties to eliminate or, at least, to avoid them. All that aiming to be able to obtain and to exploit the success in the learning, and to advance future prospective elements, to be able to achieve a double didactic success: constructive and effective.

On the other hand we cannot lead out the limitation of this work, that includes the design and the application, but not the results of the didactic sequence implementation. Therefore, the evaluation is not shown. For this reason, the validation of its effectiveness is considered as the immediate and obligatory prospective. The applied pre-experimental design has required a method that will be presented in a next publication.

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