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Defining Spatial Reasoning: A Content Analysis to Explicate Spatial Reasoning Skills for Early Childhood Educators' Use

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Abstract: Spatial reasoning is critical for mathematics learning and achievement, and its comprising skills are necessary in science, technology, engineering, and mathematics careers. To support young children in learning to reason spatially, clear definitions of the construct and supports for early childhood educators to teach the skills are needed. This study defines spatial reasoning as a comprehensive, comprehensible framework of skills. Using problem-driven content analysis, 835 text units from 103 sources, plus definitions from two reputable dictionary sources, were used to adopt, adapt, and infer the definitions for 40 terms that collectively represent spatial reasoning. Findings provide both the definitions and evidence of the extent to which various spatial reasoning skills have been investigated empirically. Directions for future research are discussed, including the need to refine the framework to ensure its utility for teachers and researchers.

Keywords: *Spatial Reasoning; Content Analysis; Mathematics Education; Early Childhood Education.*
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Introduction

Spatial reasoning is a set of uniquely human skills that allow individuals to visually recognize and mentally manipulate objects' physical properties and the spatial relations between them (Bruce et al., 2017; National Research Council [NRC], 2006). Although that general description is straightforward, defining the underlying construct is not (Lohman, 1979; McGee, 1979; Uttal et al., 2013). Current recommendations from the National Council of Teachers of Mathematics (NCTM) and the National Association for the Education of Young Children (NAEYC) indicate that mathematics instruction in early childhood classrooms should aim to develop spatial thinking skills (NAEYC & NCTM, 2010), and while spatial reasoning can develop through informal learning, formal learning opportunities strengthen the skills (Uttal et al., 2013). However, spatial reasoning has not been defined in a way that supports early childhood educators (ECEs) in teaching its comprising skills through mathematics (Pinilla, 2023), nor are ECEs adequately prepared or supported in doing so (Gilligan-Lee et al., 2022; Ginsburg et al., 2006; Moss et al., 2015).

It is well established that spatial skills (e.g., orienting, rotating, visualizing) are linked with mathematics achievement (Mix & Cheng, 2012; Sorby & Panther, 2020). For example, Gunderson et al. (2012) found that mental rotation skills at age 5 related directly to children's calculation abilities at age 8. Sorby and Panther (2020) also found strong associations between high school students' spatial skills and their mathematics scores on the Program for International Student Assessment, demonstrating that the relation persists. Further, as the skills are necessary for those in science, technology, engineering, and mathematics (STEM) careers (Lord & Rupert, 1995; Wai et al., 2009), young children need to receive spatial reasoning instruction to enhance their knowledge, skills, and abilities in this critical skill set that spans STEM domains and supports other content areas, like geography and social studies (Clements & Sarama, 2011; Jo & Bednarz, 2014; Mohan & Mohan, 2014; Newcombe, 2017). For these reasons, it is essential that ECEs learn to teach the skills; however, to help ECEs teach spatial reasoning skills, it is necessary first to state clearly what those skills are.

This article builds upon the extant research to define the spatial reasoning construct comprehensively and cohesively for use by researchers and educators, specifically in the United States (U.S.), where education standards do not specify spatial reasoning as part of the intended curriculum (Gilligan-Lee et al., 2022; Porter et al., 2011). While ECEs in other countries may be better supported in teaching these skills, definitions for most skills are implicit in the literature and, therefore, challenging to teach without support via such standards. The literature review grounds the need for such definitions based on known associations between spatial reasoning skills and mathematics learning by summarizing current conceptual frameworks of spatial reasoning and specifying the disconnect between the need, ECEs' mathematics teaching preparation, and the intended curriculum (i.e., education standards) in the U.S. A problem-driven content analysis (Krippendorff, 2019) was used to define the skills comprising spatial reasoning, which resulted in definitions for each skill presented in the focal model of spatial reasoning (i.e., Davis et al., 2015). Findings provide definitions that could be used to learn how spatial reasoning exists within early grades mathematics standards to support ECEs in teaching the skills. Findings also illustrate the disparity in frequency with which various skills have been empirically studied.

Literature Review

Spatial reasoning is ubiquitous in daily life, as the skills are necessary to operate in and interact with our physical world (NRC, 2006). Formal opportunities to learn spatial reasoning in early childhood are critical, as children taught to reason spatially experience increased mathematics achievement and STEM career interests (Guay & McDaniel, 1977; Lord & Rupert, 1985; Wai et al., 2009). Further, children with advanced spatial reasoning skills can access novel problem-solving strategies when approaching new challenges (Casey & Fell, 2018). Despite spatial reasoning's criticality in early learning (Clements & Sarama, 2011), many ECEs may need more preparation and support to teach these skills (Gilligan-Lee et al., 2022; Ginsburg et al., 2006).

Prior studies of ECEs' mathematics teaching preparation indicated they receive little to no training to incorporate spatial reasoning into their mathematics teaching practices (Clements & Sarama, 2011; Parks & Wager, 2015); however, professional learning opportunities and educational standards could support ECEs in teaching these skills. While education standards and research in Australia (e.g., Lowrie et al., 2018) and Canada (e.g., Moss et al., 2015; Hawes et al., 2017) explicitly guide ECEs to teach spatial reasoning through mathematics, similar work has not proliferated the U.S. This research represents a critical step in preparing and supporting ECEs in the U.S. by defining spatial reasoning and its comprising skills in a comprehensive, comprehensible way to support their teaching practices.

According to Gilligan-Lee et al. (2022), curricular change is needed to increase the presence of spatial reasoning in children's educational opportunities. In Ontario, Canada, spatial skills have been added to the education standards, creating a "gateway to success in STEM domains" (Gilligan-Lee et al., 2022, p. 10). In the U.S., however, such changes have not transpired. While widely adopted standards (e.g., the Common Core State Standards for Mathematics [CCSS-M]; National Governors Association [NGA], 2010) do not mandate a written curriculum, they set the goals for what children should master at each grade level. With spatial reasoning ostensibly absent from the standards, it stands to reason that it is also largely absent in ECEs' written curricula (Davis et al., 2015; Gilligan-Lee et al., 2022). Thus, ECE must incorporate the skills for young children to experience learning opportunities not explicitly represented within the standards or most written curricula, including spatial reasoning (Boykin & Noguera, 2011; Gilligan-Lee et al., 2022). To do so, ECEs need to know what spatial reasoning is and have the tools and supports to teach the related skills. Therefore, it is necessary to define spatial reasoning in a comprehensive, comprehensible format to understand how it is represented in the education standards and what teaching practices engender spatial learning opportunities. This literature review provides extant research on construct definitions of spatial reasoning and its comprising skills.

Current Spatial Reasoning Construct Definitions

Defining the spatial reasoning construct is complicated, as its content and descriptors vary between disciplines. Researchers from cognitive science, mathematics, neuroscience, physics, and psychology all study spatial reasoning but use different descriptors in their work, including spatial ability, spatial intelligence, spatial perception, spatial sense, and spatiality (Bruce et al., 2017). These fields' foci are vast and do not neatly correspond to school disciplinary content (e.g., language arts, math, science, social studies, etc.). Hence, applying many descriptions or definitions requires interpretation for educators to provide meaningful learning opportunities to young children.

This necessity of an interpretive lens highlights a challenge for ECEs in the classroom, wherein spatial reasoning may be viewed narrowly as related to geometry. From a perspective related explicitly to mathematics education and young children, Copley (2010) described thinking spatially as “visualizing objects in different positions and imagining their movements” (p. 107). While this description most closely aligns with the mathematics domain of geometry, through which visualization and other spatial skills can be taught, spatial reasoning is broader than the geometry taught in early childhood classrooms (e.g., identifying, naming, and transforming shapes; Clements & Sarama, 2011; Sinclair & Bruce, 2015).

When children work through geometric transformations (e.g., rotating, translating), they employ spatial reasoning, and we see direct connections to mathematics (e.g., the content of geometry standards in the CCSS-M; NGA, 2010). However, some spatial reasoning skills, like scaling and mapping, may be taught through social studies wherein there are implicit relations between spatial reasoning and the content domain. While the imagined or physical movements described in the first example relate to school geometry (i.e., “the study of those spatial objects, relationships, and transformations that have been formalized”; Clements & Battista, 1992), the skills set out in the second example may or may not appear in school geometry. The difference between the direct and implicit opportunities to teach spatial reasoning indicates the need for a framework that clearly defines spatial reasoning skills so ECEs might better understand the skills and how to teach them through mathematics more generally. I, therefore, sought to represent spatial reasoning through a framework that could support ECEs' mathematics instruction.

The models considered when framing this study included (a) the van Hiele levels of geometric thought (Fuys et al., 1984), (b) Uttal et al.'s (2013) classification of spatial reasoning skills, (c) recent developmental learning progressions (Perry et al., 2020; Sarama & Clements, 2009), and (d) a representation of skills that make up spatial reasoning as an integrated construct (Davis et al., 2015). Together, these models informed the development of this study's spatial reasoning conceptual framework.

As a foundation, the van Hiele levels of geometric thought (Fuys et al., 1984) appear attractive when first conceptualizing how children develop spatial reasoning skills because they show linear growth. Much work on geometry learning specific to two-dimensional shapes was driven by researchers seeing utility in van Hiele's theory (e.g., Clements & Battista, 1992), meaning their contributions pushed the field forward (Mulligan, 2015). However, the inflexible ordering of discrete skills (i.e., lockstep development of skills to describe, analyze, and make deductions about shapes) depicts only how children learn descriptive geometry and fails to capture their wealth of informal spatial reasoning knowledge (Clements, 2004b; Clements et al., 1999; Kalyankar, 2019).

Uttal et al.'s (2013) meta-analysis of spatial training studies provided a broader view of spatial reasoning in which researchers developed a classification matrix of spatial reasoning skills. Uttal et al. (2013) proposed that spatial reasoning occurs in static and dynamic contexts and can be intrinsic or extrinsic to objects (i.e., occurring to a single object or involving multiple objects and the spaces between them). While the resulting framework sufficiently described two primary constructs (i.e., spatial visualization as intrinsic spatial reasoning and spatial orientation as extrinsic), findings were directed toward researchers. In its current form, Uttal et al.'s (2013) framework is useful for education researchers developing professional learning but may not adequately support ECEs teaching spatial reasoning (Newcombe, 2013).

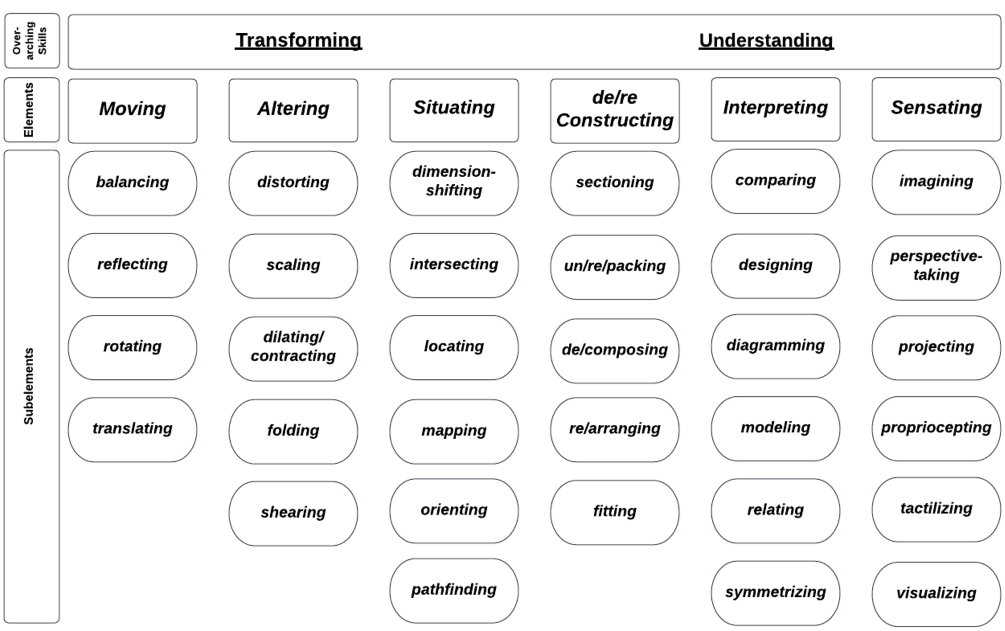
Considering theories of learning illustrates how the knowledge and skills represented in Uttal et al.'s (2013) classification matrix may develop nonlinearly (Clements, 2004a; Clements et al., 1999; Duschl et al., 2011). Specifically, developmental and learning progressions describe flexible pathways children may follow when acquiring and demonstrating spatial reasoning skills (Confrey et al., 2014; Duschl et al., 2011). Sarama and Clements's (2009) developmental progressions of early spatial reasoning skills include spatial orientation, spatial visualization and imagery, shapes, and composing two- and three-dimensional shapes; these skills have been well-researched and support the geometry strand of their prekindergarten mathematics curriculum, Building Blocks (Clements & Sarama, 2007). However, their model focuses primarily on shapes and geometry and does not capture the breadth of spatial reasoning components that would support students' mathematics learning, such as locating, mapping, and pathfinding (Davis et al., 2015). Connecting more directly to Uttal et al.'s (2013) classification, Perry et al.'s (2020) spatial reasoning learning progression delineates skills for reasoning spatially within and between objects (i.e., intrinsically and extrinsically) in static and dynamic contexts. However, Perry et al.'s (2020) learning progression and associated tools are not yet readily available through peer-reviewed publications. Whereas uptake of learning progression use would logically be high, educational practitioners generally teach from standards-aligned curricula (Kalyankar, 2019) that connect to learning progressions incompletely (Confrey et al., 2014). In other words, they support ECEs insofar as how the learning occurs developmentally but do not explicitly appear within curricula or standards.

A final model, which illustrates a holistic view of spatial reasoning and describes the breadth of the construct, is Davis et al.'s (2015) emergent representation of spatial reasoning. While it speaks clearly to the messiness of the construct and rebuffs neatly categorizing skills to articulate their interconnectedness, the terms it uses to describe the skills are not defined in spatially specific ways. For example, the word "interpreting" is used to name a skill in the model, within which subskills of "comparing" and "modeling" are meant to give examples. However, meanings of and relations between skills require inference because the terms are not defined.

For this study, I adapted Davis et al.'s (2015) representation into three levels of skills: *overarching spatial reasoning skills* (*Transforming* and *Understanding*), which hold essential spatial reasoning *elements* (i.e., A*ltering*, *de/re/Constructing*, *Interpreting*, *Moving*, *Sensating*, and *Situating*) that are each comprised of *subelement* skills (see Figure 1 for the structure). Davis et al. (2015) categorized some skills as "emergent," meaning they could fit within any of the elements; in this study's framing, they are subsumed within their most closely aligned elements. I also adapted the mathematical precision of some terminology, like updating the term *sliding* to *translating.*

Figure 1

Spatial Reasoning Conceptual Framework Structure



Note. Terms from Davis et al.'s (2015) emergent representation of spatial reasoning are shown in the structure of this study's conceptual framework of spatial reasoning.

It is important to note that the terms within the spatial reasoning conceptual framework (see Figure 1; Davis et al., 2015) are used throughout this study in specific ways. When any of the 40 terms are used as a spatial reasoning skill, they are presented in italics because many have multiple, nonspatial meanings (e.g., *moving, Understanding*, etc.). Overarching skills (i.e., *Transforming* and *Understanding*) will be called out as such; they are nebulous and not assigned into nested structures. The six spatial reasoning elements all contain subelements and appear far more frequently. The groups are demarcated with elements being capitalized and subelements in all lowercase (e.g., *Interpreting* contains *comparing, diagramming*, etc.). While the categories may seem blurred due to their inherent interrelations, subelements comprise an element, and elements constitute the overarching skills.

In specifying this study's focal model and adapting Davis et al.'s (2015) emergent conceptualization (see Figure 1), I also adopted Bruce et al.'s operational definition of spatial reasoning: "the ability to recognize and (mentally) manipulate the spatial properties of objects and the spatial relations among objects" (2017, p. 146). Although Davis et al. (2015) argued that spatial reasoning is "an emergent phenomenon . . . that cannot be fully comprehended by reducing it to its components" (p. 140), they named individual spatial reasoning skills. I agree that focusing exclusively on discrete components may detract from nuances inherent to spatial reasoning. However, there is value in defining the comprising skills, which is the focus of this paper, to support ECEs in teaching spatial reasoning through mathematics.

Research Purpose and Question

Although the importance of learning spatial reasoning skills is well established (Hawes & Ansari, 2020; Mix & Cheng, 2012; Wai et al., 2009), the skills subsumed within the construct have not been defined in a way that supports ECEs teaching them through mathematics without structural supports like education standards (Pinilla, 2023). The extant literature on spatial reasoning is aimed toward education researchers rather than teachers for classroom use (e.g., Uttal et al., 2013), and expectations for K-2 ECEs to teach spatial reasoning via educational standards as the intended curriculum is unclear (Gilligan-Lee et al., 2022; Porter et al., 2011). Therefore, this research aims to define spatial reasoning in a way that is useful to and usable by ECEs by answering the question: *How do extant descriptions and definitions of spatial reasoning skills inform a comprehensive spatial reasoning conceptual framework?*

In responding to this question, the study adds to the literature by offering a well-defined set of skills that researchers and practitioners can use as a common language when developing methods for teaching and learning through a comprehensive and comprehensible representation of spatial reasoning skills.

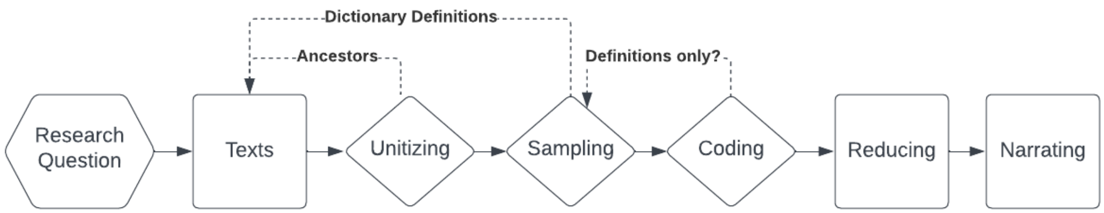
Methods

Content Analysis

I used problem-driven content analysis (Krippendorff, 2019) to address the longstanding research problem that spatial reasoning is a complex construct previously defined in ways less than accessible to educators who should be teaching the skills to children (Lohman, 1979; McGee, 1979' Uttal et al., 2013). I began with a set of terms that name discrete spatial reasoning skills (see Figure 1; Davis et al., 2015) as a priori categories that I iteratively defined based on the extant literature using Krippendorff's (2019) content analysis logic model. Figure 2 provides an overview of the content analysis phases and illustrates the recursive, interactive nature in which this content analysis was approached.

Figure 2

Content analysis methods overview



Note. Krippendorff's (2019) content analysis components adapted to include recursive data collection and simultaneous data analysis. The flow chart begins with the research question. Unidirectional processes and recursive decision points are then demarcated as rectangles and rhombuses, respectively.

The overview (see Figure 2) specifies the research question as the origin and progresses through the components sequentially with recursive looping to respond to the research problem. The components (i.e., unitizing, sampling, coding, reducing, inferring, and narrating; Krippendorff, 2019) and the looping occurred through analysis cycles so I could reduce the data to broad yet salient definitions that I also represented as a conceptual framework of spatial reasoning. That is, while the research components appear discrete and rigidly structured, moving recursively between the components is supported so long as trustworthiness, reflexivity, or other means to express the authenticity of findings are present within the analysis and its outcomes (e.g., a formative audit trail; see excerpts in Appendix A; Denzin & Lincoln, 2000; Krippendorff, 2019).

To illustrate objectivity and systematicity in my methods, the text source selection process is next described. I then review each component named in Krippendorff's (2019) model, as outlined in Figure 2, noting where spiraling between components occurred during data collection and analysis (Creswell & Poth, 2018).

Data Collection

Data collection included selecting texts and unitizing meaningful text units across sources (see the first two processes in Figure 2). I selected primary sources by reviewing the models of spatial reasoning detailed in this study's literature review. To ensure that sources were relevant to the current educational context in the U.S., wherein teaching practices are heavily influenced by education standards that do not specify spatial reasoning as a target goal in mathematics, I selected only texts written within the last 40 years, as standards-based education reform conceivably began in 1983 (i.e., aligns with the publication of *A Nation at Risk*, by the National Commission on Excellence in Education, 1983). While this led to the exclusion of the translated version of the van Hiele levels of geometric thought, as the original research was written in Dutch in 1957 (Fuys et al., 1984), I added the NRC's (2006) authoritative text on spatial reasoning in education, *Learning to Think Spatially*, due to its focus on incorporating spatial reasoning instruction into schools. Table 1 contains a detailed rationale for each primary source's inclusion.

Table 1

Primary Source Text Selection

Source	Rationale
Spatial Reasoning in the Early Years: Principles, Assertions, and Speculations (Davis & the Spatial Reasoning Study Group, 2015)	This edited book emphasized the importance of spatializing school curricula in early grades and was selected because this study's spatial reasoning construct definition was adapted from Davis et al.'s (2015) emergent representation of spatial reasoning in the text's concluding chapter.
Early Childhood Mathematics Education Research: Learning Trajectories for Young Children (Sarama & Clements, 2009)	In this authored book, Chapters 7, 8, and 9 are dedicated to spatial thinking and recapitulated decades of research on young children's mathematical development. It was selected for its comprehensiveness in describing young children's spatial reasoning development. The book was used as a primary source text to develop Perry et al.'s (2020) spatial reasoning learning progression.
"The Malleability of Spatial Skills: A Meta-Analysis of Training Studies" (Uttal et al., 2013)	This meta-analysis of spatial training studies was included due to the broadly applicable conceptual framework that emerged to classify spatial reasoning skills (i.e., the 2 x 2 classification of spatial reasoning skills: intrinsic static, intrinsic dynamic, extrinsic static, and extrinsic dynamic skills).
Learning to Think Spatially (National Research Council, 2006)	This edited report connected the importance of spatial thinking to 21st-century skills and career readiness. Many spatial thinking components are specified and defined, as well as paths forward for teaching spatial reasoning in schools.

The primary sources were used for the first round of data collection. When unitizing text, I selectively included text units at the sentence level for the rich details they provided about each skill. I extracted units that either explicitly defined the term or contained it in a way meaningful to spatial reasoning and stored them in an Excel workbook. After collecting units from all primary sources, I identified those that had citations, recursively located their ancestral sources, and repeated the unitizing process as second-round data collection (see the recursive loop in Figure 2; a complete list of ancestral sources is found in Appendix B). I did not extend the search beyond first-level ancestors to bound data collection and ensure the corpus remained manageable (Krippendorff, 2019). However, I added definitions from two online dictionary sources (i.e., Merriam-Webster, n.d.; Oxford University Press, 2022) as third-round data collection after discovering the need to clarify terms when coding sampled units from the primary and ancestral texts. In total, I sampled 835 text units from 103 sources, of which 522 were from primary sources, and 283 units emerged from ancestral sources.

However, developing units of analysis about terms within this complex construct was inherently messy, as scholars in different fields (e.g., cognitive science, mathematics education, philosophy, and psychology; Bruce et al., 2017) used different words or phrases to describe similar concepts. This was prevalent for words used frequently within colloquial speech (e.g., *Transform*, *Understand*), two-word phrases (e.g., *dimension-shifting, perspective-taking*), and terms only mathematically precise in context (e.g., *reflecting*). When I located colloquial terms in ways unrelated to spatial reasoning, I did not unitize their mention. For two-word phrases, I only captured units that described the term holistically. For example, I did not unitize text that only described dimensions when unitizing text for *dimension-shifting* because the unit needed to specify that there was a change. Interestingly, I expected to unitize the word "reflect" frequently but found few mentions. Based on my background knowledge of early mathematics, I replaced the search term with "flip" as an analytic lens (Krippendorff, 2019) and expanded the data set fivefold. This update allowed me to analyze a more substantial body of text when deriving a meaningful definition for *reflecting*. Across these instances and others, I created memos to support coding and content reduction (Saldaña, 2016).

Data Analysis

I next completed the sampling, coding, reducing, abductively inferring, and narrating components of content analysis (see Figure 2; Krippendorff, 2019) to adopt, adapt, and infer definitions. I exported the unitized text into NVivo 1.6 (https://lumivero.com/products/nvivo/) and sampled the most informative units. Whereas content analyses traditionally employed statistical sampling techniques to ensure the analysis was based on sufficient information, I used purposive and snowball sampling (Krippendorff, 2019) to ensure each unit contributed to an understanding and definition of the spatial reasoning terms.

While sampling, I found that most words (i.e., 33 of 40 terms) had no explicit definitions within the corpus. To triangulate across sources and meet the need to define spatial reasoning in a way that would be useful to and useable by teachers, I added definitions for all terms from two reputable online dictionaries: Merriam-Webster (n.d.) and the Oxford English Dictionary (Oxford University Press, 2022). I did not add definitions for two-word terms (i.e., *dimension-shifting, perspective-taking*), as the definitions for the individual words did not inform the data corpus meaningfully. Such iterative looping between data collection and analysis (Creswell & Poth, 2018) was critical to developing broadly applicable yet spatially meaningful definitions; I documented my decisions and tacit reasoning in my audit trail to enhance the findings' trustworthiness, support the methods' replicability, and reduce bias (see Appendix A; Koch, 1994; LeCompte, 2000; Lincoln & Guba, 1985).

After sampling informative units, I holistically coded the corpus using provisional codes based on the overarching skills, elements, and subelements (see Figure 1) to organize all data across sources in NVivo (Saldaña, 2016). I coded the data associated with elements (i.e., *Altering, de/re/Constructing, Interpreting, Moving, Sensating, Situating*) as parent codes and the data associated with their subelements as child codes. This allowed data within the child codes to support the development of definitions for the parent codes in their nested node structure within NVivo (Miles et al., 2014; Saldaña, 2016). I winnowed the provisionally coded text using constant comparison between the units and spatial reasoning construct definition to ascertain whether each unit was relevant to defining its associated term (Glaser & Strauss, 1967; Merriam, 1998; Saldaña, 2016).

I next reduced the coded text using constant comparison methods (Glaser & Strauss, 1967; Merriam, 1998) to narrate definitions. When explicit definitions were located in a primary or ancestral source, other text units were generally excluded from further analysis. If I found only one definition and it was broadly applicable, I adopted it. However, I located multiple definitions for most words with explicit definitions; those were then the only units from which I inferred a final meaning. To illustrate the use of constant comparison (Glaser & Strauss, 1967; Merriam, 1998), Table 2 includes the definitions of *visualizing* from extant literature. Moving from left to right, I highlighted similar phrasing across the definitions, wrote memos to understand the definitions' common meanings, and synthesized the definition that was ultimately included in the resulting conceptual framework.

Table 2

Example of Constant Comparison Methods

Source	Text units defining *visualizing*	Memos	Synthesized definition
Arcavi, 2003, p. 217	The ability, the process, and the product of creation, interpretation, and the use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas, and advancing understandings	Focused on an ability Consistently considers mental manipulation, which requires first holding the object in the mind's eye (i.e., imagining) Perceptions and spatial patterns mentioned sometimes *Process mentioned some—focus in here for action	Imagining and mentally transforming spatial representations
Chu & Kita, 2011, p. 102	The ability to mentally transform complex stimuli (e.g., three-dimensional object) in space		
French et al., 1963, p. 47	The ability to manipulate or transform the image of spatial patterns into other visual arrangements		
Hegarty & Waller, 2006, p. 127, as cited in Chu & Kita, 2011, p. 102	The ability to mentally manipulate, rotate, twist, or invert objects without reference to oneself		
Mix & Cheng, 2012, p. 200	The ability to perceive complex spatial patterns and comprehend imaginary movements in space		
NCTM, 2000, p. 41	Building and manipulating mental representations of two and three-dimensional objects and perceiving an object from different perspectives		
Presmeg, 1997, p. 304	The process involved in constructing and transforming visual mental images, as well as those used in drawing figures or diagrams or constructing or manipulating them on computer screens		

Note. The comparisons in this example are between explicit definitions for *visualizing*. The process was similar whether coded units were definitions or descriptions.

This example shows methods for defining a skill with existing definitions; similar methods were used for skills with descriptions only. Following this closer coding process, I reduced the diverse body of text to infer final meanings abductively (Krippendorff, 2019) by printing the coded text from NVivo and hand-coding it for the last reduction cycle. I drew inferences by considering the skills as relevant to spatial reasoning and meaningful to one another within their nested structure to narrate final definitions. See Appendix C for the complete definitions.

Findings

In this study, I used problem-driven content analysis (Krippendorff, 2019) to examine the extant literature systematically and comprehensively define the 40 skills comprising spatial reasoning (See Figure 1; Davis et al., 2015). Table 3 delineates the number of primary and ancestral sources from which I unitized text and the counts of units by type, as counts of mention (i.e., unitized text), counts of definitions (i.e., explicit definitions), and counts of coded textual elements (i.e., coded text). I also calculated descriptive statistics to depict the variation found when unitizing this body of literature and sampling units for coding (see Table 4). Findings are described with reference to data in these tables together. Despite the superficial nature of using counts to describe content analysis findings (Krippendorff, 2019), it was the first practical step to describe the unitizing and sampling outcomes, as the representation of spatial reasoning terms in the extant literature varied widely. In this section, I provide the results of the quantitative analysis and the consequent definitions.

The source counts shown in Table 3 describe the prevalence of each term within this body of literature. For example, I unitized text for the *Situating* subelements *orienting* and *pathfinding* from all four primary sources, but they were not equally represented in the larger field of literature. I found informative units of text for *orienting* in 11 ancestral sources, whereas I only found units that meaningfully described *pathfinding* in two. I unitized text from 2.88 primary sources per term on average, but most frequently sampled units from three (M = 2.88, Mo = 3; see Table 4.2). In 12 instances, I sampled units from all four primary sources, and in one instance, I sampled units from none; *shearing* was only used in Davis et al. (2015), but never in a way that gave it explicit meaning. Similarly, on average, I unitized text from 3.75 ancestral sources per term, but frequently, no ancestral sources meaningfully informed definitions of the terms. Therefore, I added units from online dictionaries to surmise a definition, *shearing* included.

The source counts also show that some spatial reasoning skills have been researched far more frequently than others (i.e., spatial orientation and visualization). Specifically, *visualizing* and *orienting*, often considered core spatial reasoning abilities, have been defined by numerous researchers (e.g., Hegarty & Waller, 2005; Mix & Cheng, 2012; see counts in Table 3); their definitions were similar across sources and informed my adapted definitions without the need for additional unitized text. However, I located existing definitions for less than 20% of the included spatial reasoning skills (see Tables 3 and 4).

Table 3

Frequency Counts for Content Analysis

Spatial reasoning term	Source counts		Counts by type		
	Primary	Ancestor	Unitized text	Explicit definition	Coded text[a]
Overarching skills					
Transforming	4	12	51	4	4
Understanding	4	4	41	0	3
Elements & subelements					
Moving	3	1	17	0	4
balancing	2	1	6	0	5
reflecting (flipping)	2	0	10	0	3
rotating (turning)	4	8	43	2	2
translating (sliding)	2	4	13	0	6
Altering	3	0	4	0	6
distorting/morphing	3	2	10	0	8
scaling	3	9	38	3	3
dilating/contracting	2	0	2	0	4
folding	3	5	16	0	7
shearing	1	0	2	0	3
Situating	2	0	3	0	5
dimension shifting	2	1	4	0	2[b]
intersecting	3	0	7	0	3
locating	4	12	56	0	14
mapping	3	4	25	0	11
orienting	4	11	50	2	8
pathfinding	4	2	17	0	7
de/re/Constructing	3	6	40	0	9
de/re/arranging	3	4	17	0	8
de/re/composing	3	1	15	0	4
fitting	3	0	7	0	5
packing	1	0	4	0	5
sectioning	2	1	7	0	3
Interpreting	3	8	40	1[c]	9
comparing	4	4	30	0	11
diagramming	4	7	28	0	15
modeling	3	9	34	0	6
designing	2	1	6	0	3
relating	4	9	55	0	18
symmetrizing	3	5	26	0	8
Sensating	1	0	1	0	3
imagining	4	3	27	0	9
perspective-taking	4	3	26	1[d]	1[d]
projecting	2	0	9	0	3
propriocepting	2	2	4	0	3
tactilizing	3	0	7	0	3
visualizing	4	11	37	7	7
Total	-	-	835	20	241

Note. [a] Counts of "coded text" may be greater than the counts of unitized text because they include two dictionary definitions. [b] Two-word terms without dictionary definitions (e.g., *dimension-shifting*) did not have the increase in coded units unless definitions for individual words (e.g., dimension and shifting, separately) meaningfully contributed to defining the term. [c] Explicit definitions that described skills too narrowly (e.g., *interpreting* only within the context of interpreting a map) informed synthesized definitions but were not directly adopted. [d] The definition was considered sufficient for the term with one broadly applicable definition (i.e., *perspective-taking*), and coding was discontinued.

Table 4

Descriptive Statistics for Phase 1 Content Analysis

Variable	Mean	Median	Mode	Range (min–max)
Source counts				
Primary	2.88	3.00	3	0–4
Ancestor	3.75	2.50	0	0–12
Counts by type				
Unitized text	20.88	16.50	4, 7	1–56
Explicit definition	0.50	0.00	0	0–7
Coded text	6.03	5.00	3	1–18

The counts by unitized text, explicit definitions, and coded text together illustrate the winnowing process that happened through the analysis. Considering the counts of unitized text, I sampled 20.88 units on average, though eight terms had 40 or more units, which heavily influenced the mean (see Table 4). Most frequently, fewer than 17 units were sampled (*Mdn* = 16.5). There was considerable variation between the number of units sampled by term. For example, I collected 56 units to derive a definition for *locating*, yet I collected only one unit to inform the definition of *Sensating.* Of the seven terms with explicit definitions located, two had only one definition, while one (*visualizing*) had seven; the other 33 terms had none (see Table 3). Finally, the counts of coded text in the right column of Table 3 indicate how many units were used to adopt, adapt, or abductively infer a definition from extant descriptions. I next describe the findings by terms with explicit definitions and those without.

Skills With Explicit Definitions

I first focused on locating and comparing skills' extant definitions and found explicit definitions for only seven of the 40 spatial reasoning terms. I found and adopted a single, broadly applicable definition for one term (*perspective-taking*; Muir & Cheek, 1986). For four others (*rotating*, *scaling*, *Transforming*, and *visualizing*), I located multiple definitions and synthesized them through open coding and constant comparison (Merriam, 1998; Saldaña, 2016). The existing definitions for these terms generally aligned in meaning, and I found value in using language from each. For instance, when defining *visualizing*, I examined the similarities and differences between seven extant definitions to derive one, retaining the most common themes within a broader synthesized definition (see example in Table 2).

I altered my methodology and used additional text units for the remaining two skills with existing definitions (i.e., *interpreting* and *orienting*). The explicit definition for *interpreting* was narrowly applicable (i.e., focused on interpreting maps), so I included other units to infer its final definition. While I located two definitions of *orienting*, many descriptions were informative. For example, Uttal et al. (2013) stated that spatial orientation "involves the ability to imagine oneself or a configuration from different perspectives" (p. 353). Given the extensive research on spatial orientation, I inferred this excerpt as meaningful to crafting *orienting*'s definition because the two definitions I located were unlikely the only ones that existed, and other sources warranted consideration.

Abductively Inferred Definitions

Definitions for the other 33 spatial reasoning skills were based on coded and reduced unitized text, with attention to text that communicated ideas found redundantly across sources. For example, when defining *comparing,* I used text from Lowenstein and Gentner (2001), who described *comparing* "as a structure-mapping process that . . . acts to render common relational structures salient" (p. 192), and from Sarama and Clements (2009), who said that *comparing* could be "represented as a mapping between sets of relations between components of the representations" (p. 223). The idea of mapping between representations to make sense of spatial relations was found in both and informed the definition (i.e., Judging sameness or difference by distinguishing between forms using appearance-based relational reasoning; mapping correspondences between two or more forms; see Appendix C). Similar methods were used to consider information across sources when abductively inferring the remaining definitions.

After adopting, adapting, or inferring the definitions, I narrated them as a nested text list; see Appendix C for the definitions and the sources used in creating them.

Discussion

This study was conducted to set the foreground for future research on supporting ECEs to teach spatial reasoning through mathematics. Before supporting ECEs, I needed to articulate what spatial reasoning is, as existing definitions of the construct and its comprising skills needed refinement for practitioner use (Hegarty & Waller, 2005; Uttal et al., 2013). I used content analysis methods to synthesize a large body of extant research into broadly applicable definitions of spatial reasoning skills to support ECEs in teaching them through mathematics (see Appendix C). This explication of the terms resulted in a conceptual framework that fills gaps in current models. That is, to address the ambiguity of what spatial reasoning is, I adapted and defined the terms Davis et al. (2015) presented as the "emergent complexity of spatial reasoning" (p. 140) as a conceptual framework that could be integrated into ECEs' professional learning and teaching practices.

The skills for which I found only descriptions and not definitions (N = 33) varied between novel descriptors (e.g., *dimension-shifting, pathfinding,* or *propriocepting*) and words that are omnipresent in our daily speech. One example of the latter is *Understanding*, which is a spatial reasoning skill and a word frequently used in nonspatial ways. Merriam-Webster (n.d.) defined "understand" as "a mental grasp" or "comprehension." In contrast, the framework developed in this research associates it more closely with spatial reasoning: realizing and making sense of [spatial] relationships (see Appendix C). The differences are nuanced but critical, given Hegarty and Waller's (2005) reiteration of McGee's (1979) point that a primary challenge in defining a spatial ability stemmed from the inconsistent language used by researchers. This study's resulting spatially relevant definitions could facilitate researchers and practitioners in speaking the same language when discussing specific skills. While these definitions supplement the existing models, the variation in amounts of extant literature describing each term indicates that more refinement may be needed.

The terms used to name spatial reasoning skills, their granularity, and potential overlaps between those skills serve as limitations to this study. Specifically, I used Davis et al.'s (2015) conceptualization of spatial reasoning as the focal model to develop the framework, which used some unique descriptors. While I made some alterations to the terminology (e.g., changing "sliding" to "translating") and structure of terms' nesting (i.e., incorporating the "emergent competencies" into the elements) when developing the definitions, other skills from Davis et al.'s (2015) model (e.g., *propriocepting*, *Situating*, and *tactilizing*) were retained. Given the infrequency with which some skills were described in the extant literature, further work is needed to understand what the core spatial reasoning elements are to determine if these are distinct to Davis et al. (2015) or simply less developed from the literature.

Early feedback on the conceptual framework indicated that the number of terms could be too granular to have utility for teachers and mathematics education researchers. Similarly, there were questions about how some skills potentially overlap, like *decomposing and sectioning*. These might be too similar to parse from one another meaningfully. With these findings and limitations in mind, meaningful implications and directions for future research emerged.

Implications

This study's resulting definitions of spatial reasoning skills and conceptual framework offer tools to communicate what spatial reasoning is and examine how its comprising skills appear in teaching practices, curriculum, and professional learning opportunities. While the framework provides broadly applicable definitions that can be refined moving forward, iteration is needed to make it useful for most teachers and researchers. I anticipate that through future research and the development of curricular supports, this tool could help ECEs find avenues to teach spatial reasoning through mathematics and beyond. However, to support ECEs in using this framework as a tool, opportunities should be provided for them to learn about spatial reasoning and how to connect its comprising skills to their current instructional practices, whether through school geometry, other mathematics content, or other content domains. Together, these implications of iteratively revising the framework and providing ECEs with professional learning opportunities provide directions for future research.

Future Research

Given the need to revise the framework and provide opportunities for teachers to learn about using this tool, there are three directions for future research. First, additional feedback is needed from mathematics education researchers with expertise in spatial reasoning. Sharing the contents and structure of the framework and soliciting feedback will be vital in providing ECEs and other educators with a comprehensive, comprehensible tool. Future research should include working with subject matter experts to revise the framework's structure and content. Focus groups with experienced ECEs would additionally support teachers' involvement in developing future versions of the framework to ensure its usability.

Second, there is a need to examine the mutual exclusivity of skills represented by each term (i.e., multiple terms in the framework might represent the same underlying skill). The framework's refinement might preempt some of this work; if skills and the terms representing them are updated or clustered when working with subject matter experts, that may reduce the work herein required. Specifically, some terms had a small literature base to support their synthesized or inferred definitions, meaning they may not be discrete skills but instead represent another skill in the framework backed by extensive research. After soliciting feedback and refining the framework, I recommend developing tasks to elicit each skill, conducting cognitive interviews (Leighton & Gierl, 2007) with children, and assessing their responses for skill overlap. Resources related to testing many of the skills are available through the Northwestern University (2023) Spatial Intelligence and Learning Center and Moss et al.'s (2016) book *Taking Shape.*

Third, as alluded to in the implications, teachers need opportunities to understand the underlying construct and the contents of the framework to use it as a tool. There is a need to develop professional learning opportunities that meet ECEs' learning and teaching needs within the contexts in which they are positioned as learners. One way to support professional learning and teaching practice is to integrate these definitions into a learning progression (Confrey et al., 2014; Duschl et al., 2011) and provide associated professional development. Such professional learning could help ECEs distinguish between the content of school geometry (Clements & Battista, 1992) and the thinking involved in spatial reasoning, including how the skills extend within mathematics learning and other domains.

Conclusion

This study's findings provide initial definitions for spatial reasoning skills and evidence of which skills have been researched more and less frequently. While we know spatial reasoning is critical for mathematics learning (Hawes & Ansari, 2020; Mix & Cheng, 2012; Sorby & Panther, 2020), it was not previously defined in a way that supported ECEs in the U.S. to teach its comprising skills. The definitions presented through this content analysis (see Appendix C) and the model as conceptualized (see Figure 1) lay a foundation for future research, which could be addressed through the directions listed.

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Appendix A

Formative Audit Trail Excerpts

Date	Data	Issue	Decision	Exceptions & updates
8/16/22	Primary sources (Sarama & Clements, 2009)	When unitizing text for each skill, how to handle mentions of the skill that do not contribute to an understanding	Read sentences before and after; if no meaning when expanding unit size, do not unitize the mention	Update: If there is a reference in the sentence, pull the ancestor and only unitize if it provides meaning
8/18/22	Ancestral sources (Bremner & Taylor, 1982)	What to do when an ancestral source is illegible or unavailable	Construct the definition without it using other sources if the available sources provide an adequate understanding	Exception: If there are fewer than three units, find the original
8/30/22	Across sources	Adopting definitions	Create a new tab in analysis workbook and put firm definitions there	Update: when multiple definitions are located, use constant comparison
9/2/22	Across ancestors	How to handle unexpected terms in ancestral sources (e.g., Chu & Kita, 2011, was pulled for *transforming* but had units for *rotating* and *folding*)	If the units are definitions, unitize them; if they are descriptions, do not	
9/10/22	Dictionaries	How to make definitions meaningful for terms that have few extant descriptions that are specialized	Add definitions from Merriam-Webster and Oxford online dictionaries for all terms	Exception: dictionary definitions not added for two-word skills unless single word definitions are meaningful
9/17/22	Across sources	Developing coding structure in NVivo	Use spatial reasoning skills as a priori codes; overarching skills, elements, and “emergent competencies” label are at the parent level—elements and emergent competencies have child codes for comprising skills	
9/28/22	All sources	Drafting definitions based on coding	Print all coded units by term from NVivo and hand code remaining data to create salient definitions	Exception: If there are explicit definitions for a term, only print that data

Date	Data	Issue	Decision	Exceptions & updates
10/10/22	All sources	Narrating definitions—what form communicates best	Create list for research, graphic form for conference presentations and discussing with others	

Appendix B

Ancestral Sources by Term

Skill	Ancestral Sources
	Overarching skills
Transforming	Amorim et al., 2006; Chu & Kita, 2011; Dehaene et al., 2006; Ehrlich & Levine, 2006; Gunderson et al., 2012; Hegarty & Waller, 2005: Just & Carpenter 1985; Kosslyn, 1983; Levine et al., 1999; Perham, 1978; Rosser et al., 1988; Schultz & Austin, 1983; Williford, 1972; Wright et al., 2008.
Understanding	Edwards, 1991; Liben & Downs, 1989; Liben & Yekel, 1996; Myers & Liben, 2008
	Elements & subelements
Moving	Clements & Burns, 2000; Krutetskii, 1976; Newcombe, 1989
balancing	Kersch et al., 2008
reflecting	--
rotating	Chu & Kita, 2011; Clements & Burns, 2000; Clements et al., 1996; Kersch et al., 2008; Krutetskii, 1976; Mix & Cheng, 2012; Perham, 1978; Roberts & Aman, 1993; Rosser et al., 1988; Sarama et al., 1996; Scholnick et al., 1990; Vasilyeva & Huttenlocher, 2004; Wheatley, 1990
translating	Kamii et al., 2004; Moyer, 1978; Perham, 1978; Radford, 2014; Schultz & Austin, 1983; Stiles, 2001
Altering	--
distorting/morphing	Bremner & Taylor, 1982; Liben & Downs, 1989
scaling	Anooshian et al., 1984; Blades et al., 2004; DeLoache, 1987; Frick & Newcombe, 2012; Hanson & Hanson, 1993; Möhring et al., 2014; Muir & Cheek, 1986; Newcombe & Huttonlocher, 2000; Siegel & White, 1975; Vasilyeva & Huttenlocher, 2004
dilating /contracting	--
folding	Chu & Kita, 2011; Empson & Turner, 2006; Newcombe & Shipley, 2014; Wheatley, 1996; Wright et al., 2008.
shearing	--
Situating	--
dimension- shifting	--
intersecting	--
locating	Bertenthal et al., 1994; DeLoache, 1987; DeLoache & Burns, 1994; Ehrlich & Levine, 2006; Geary et al., 2000; Henderson & Taimina, 2005; Huttonlocher et al., 1999; Landau, 1996; Muir & Cheek, 1986; Newcombe & Huttonlocher, 2000; Rieser et al., 1982; Vasilyeva & Bowers, 2006; Voyer et al., 2007; Wang & Spelke, 2002
mapping	Greenough et al., 1987; Liben & Downs, 1989; Muir & Cheek, 1986; Newcombe & Huttonlocher, 2000; Siegel & White, 1975; Vasilyeva & Bowers, 2006
orienting	Cheng et al., 2013; Clements & Battista, 1992; Franklin & Tversky, 1990; Hegarty & Waller, 2005; Jauåovec & Jauåovec, 2012; Mix & Cheng, 2012; Rosser, 1994; Rosser et al., 1984; Shepard & Metzler, 1971; Shepard & Cooper, 1982; Suwa & Tversky, 1997

Skill	Ancestral Sources
pathfinding	Bushnell et al., 1995; Rieser et al., 1994; Rush & Moore, 1991
de/re/Constructing	Battista et al., 1998; Casey, Andrews et al., 2008; Casey, Erkut, et al., 2008; Seo & Ginsburg, 2004; Steffe & Cobb, 1988; Taylor & Hutton, 2013; Tzuriel & Egozi, 2010; Uttal, 1996
arranging	Chase & Simon, 1973; Goodson, 1982; Sarama et al., 1996; Suwa & Tversky, 1997; Wheatley, 1990
de/re/composing	Sarama et al., 1996; Steffe & Cobb, 1988
re/un/packing	--
fitting	--
sectioning	Mamolo et al., 2011; Newcombe & Shipley, 2014
Interpreting	Arcavi, 2003; Cheng et al., 2013; Dalke, 1998; Hallowell et al., 2015; Liben & Downs, 1989; Merleau-Ponty, 1962; NCTM, 2000; Newcombe & Huttonlocher, 2000; Sarama et al., 2003; Steenpaß & Steinbring, 2013; Thom & McGarvey, 2015
comparing	Clements et al., 1999; Lehrer, 1998; Loewenstein & Gentner, 2001; Vurpillot, 1976
designing	Caldera et al., 1999
diagramming	Cariglia-Bull & Pressley, 1990; Hegarty & Just, 1993; Heiser & Tversky, 2002; Presmeg, 1997; Simon, 2001; Steenpaß & Steinbring, 2013; Stieff, 2007; Taylor & Hutton, 2013
modeling	Blades et al., 2004; Blaut & Stea, 1974; Boardman, 1990; Brosnan, 1998; Caldera et al., 1999; Cariglia-Bull & Pressley, 1990; Greenough et al., 1987; Hegarty & Just, 1993; Liben, 1988; Muir & Cheek, 1986; NCTM, 2000; Serbin & Conner, 1979; Simon, 2001; Taylor & Hutton, 2013
relating	Bryant, 2008; Case et al., 1996; Clements et al., 1996; DeLoache, 1987; Kastens & Ishikawa, 2006; Kirkwood et al., 2001; Mulligan & Mitchelmore, 2009; Newcombe, 1989; Newcombe & Huttonlocher, 2000; Newcombe & Sluzenski, 2004
symmetrizing	Bremner & Taylor, 1982; Bryant, 2008; Mackay et al., 1972; Sarama et al., 1996; Seo & Ginsburg, 2004; Uttal, 1996; Vurpillot, 1976
Sensating	--
imagining	Hegarty & Waller, 2005; Newcombe, 1989; Rieser et al., 1994; Tahta, 1980
perspective-taking	Hegarty & Waller, 2005; Muir & Cheek, 1986; Rosser et al., 1988
projecting	--
propriocepting	Newcombe & Huttonlocher, 2000; Thurston, 1995
tactilizing	--
visualizing	Arcavi, 2003; Caldera et al., 1999; Clements & Burns, 2000; French et al., 1963; Hanson & Hanson, 1993; Kosslyn, 1983; Kozhevnikov et al., 2002; Kozhevnikov et al., 2005; Krutetskii, 1976; Mix & Cheng, 2012; NCTM, 2000; Presmeg, 1997; Serbin & Conner, 1979

Appendix C

Inferred Definitions for Spatial Reasoning Framework

Skill	Operational definition†	Sources
	Overarching skills	
*Transforming**	Changing, moving, creating, or removing objects, and imagining outcomes of those changes	Chu & Kita, 2011; Davis et al., 2015; Hegarty & Waller, 2004; NRC, 2006; Newcombe & Shipley, 2014; Williford, 1972
Understanding	Realizing and making sense of spatial relationships	Edwards, 1991; Liben & Downs, 1989; Liben & Yekel, 1996
	Elements & subelements	
Moving	Changing the position of something by using spatial transformations (i.e., Slide, Flip, Turn)	NRC, 2006; Uttal et al., 2013
balancing	Bringing into proportion/visual equilibrium by creating equivalence/sameness	Davis et al., 2015; Kersh et al., 2008
reflecting	Causing something to change direction or double back (i.e., flipping)	Davis et al., 2015; Sarama & Clements, 2009
*rotating**	Turning 2- and 3-D figures	Kersh et al., 2008; Mix & Cheng, 2012
translating	Sliding objects smoothly along a surface, often in a vertical, horizontal, or diagonal trajectory	Davis et al., 2015; Moyer, 1978; Perham, 1978
Altering	Modifying or changing something's appearance (or making something different by modifying it in some way without changing what it is)	Davis et al., 2015; Sarama & Clements, 2009; Uttal et al., 2013
distorting/ morphing	Not preserving the proportion/shape when altering	NRC, 2006; Sarama & Clements, 2009
*scaling**	Transforming and understanding the correspondence of distance information from one space to another of a different size, while retaining proportion	Frick & Newcombe, 2012; Möhring et al., 2014; Vasilyeva & Huttonlocher, 2004
Dilating /contracting	Enlarging/expanding or reducing/drawing together, without changing the shape and proportion of an object	Davis et al., 2015; Sarama & Clements, 2009
folding	Spatially transforming by doubling/pleating a two-dimensional figure upon itself	Chu & Kita, 2011; Empson & Turner, 2006; Newcombe & Shipley, 2014; Uttal et al., 2013
shearing	Cutting something off or altering something as an angular shape	Davis et al., 2015
Situating	Putting something in or experiencing some place, situation, context, or belongingness to a category	Davis et al., 2015; NRC, 2006
*dimension-shifting**	Moving between two and three dimensions (e.g., shapes and their representations)	NCTM, 2000
intersecting	Meeting or crossing at a point or in a plane	NRC, 2006; Sarama & Clements, 2009

Skill	Operational definition†	Sources
locating	Using extrinsic spatial skills to find a place, context, or situation based on cues (i.e., cue learning of landmarks relative to the target object) or place (i.e., place learning by finding an object based on its coded location as distance and direction on a coordinate system); establishing knowledge of an object's site	Bertenthal et al., 1994; Davis et al., 2015; DeLoache, 1987; Huttonlocher et al., 1999; Muir & Cheek, 1986; Newcombe & Huttonlocher, 2000; Voyer et al., 2007
mapping	Using or creating sociocultural tools that represent corresponding spatial locations. • Map reading: comprehending relationships through viewpoint-specific representations (e.g., aerial vs. oblique/side view maps) • Map-making: depicting spatial relationships by identifying and recording information about object locations between spaces	Blades & Spencer, 1994; Davis et al., 2015; Liben & Downs, 1989; Muir & Cheek, 1986; NRC, 2006; Newcombe & Huttonlocher, 2000; Sarama & Clements, 2009; Uttal., 1996; Vasilyeva & Bowers, 2006
orienting	ascertaining one's bearings when acquainting with environments and perceiving the positions of various objects in space relative to one another and the viewer; also, imagining oneself or a configuration from a different perspective; understanding and operating on the relationships between the positions of objects in space with respect to one's own position	Clements & Battista, 1992; Hegarty & Waller, 2005; Mix & Cheng, 2012; Sarama & Clements, 2009
pathfinding	Navigating by finding and making a way using perceptual cues or cognitive systems	Bushnell et al., 1995; Davis et al., 2015; NRC, 2006; Sarama & Clements, 2009
de/re/ Constructing	Making or forming a spatial structure by organizing components/parts into a more complex whole; deconstructing: breaking a structure into component parts or elements; reconstructing: re-forming/making the structure in a different configuration	Battista et al., 1998; Casey, Andrews et al., 2008; Sarama & Clements, 2009; Uttal., 1996
arranging	Placing or perceiving the placement of multiple objects in physical or conceptual relational structures; rearranging: putting objects into an arrangement in a new or better way	Chase & Simon, 1973; NRC, 2006 Sarama & Clements, 2009; Suwa & Tversky, 1997
de/re composing	Putting together parts or elements to create complex units or wholes; decomposing: taking units/wholes apart into constituent parts or elements	Sarama & Clements, 2009
re/un/packing	Placing items/objects inside of something else compactly (possibly to capacity); unpacking: removing contents from something; repacking: refilling in a new, different, or more efficient way	NRC, 2006
fitting	Putting new pieces/parts into an existing configuration to fill a space	NRC, 2006; Uttal et al., 2013
sectioning	Cutting into parts (that may or may not be equal)	Newcombe & Shipley, 2014
Interpreting	Drawing inferences and/or conclusions by expounding the meaning of problems and representations (e.g., map-reading, visualizing, conjuring mental images, or seeing patterns and sense-making based on those interpretations)	Davis et al., 2015; Liben & Downs, 1989; NCTM, 2000; NRC, 2006; Sarama et al., 2003; Steenpaß & Steinbring, 2013

Skill	Operational definition†	Sources
comparing	Judging sameness or difference by distinguishing between forms using appearance-based relational reasoning; mapping correspondences between two or more forms	Clements et al., 1999; Davis et al., 2015; Lehrer et al., 1998; Lowenstein & Gentner, 2001; Sarama & Clements, 2009; Vurpillot, 1976
designing	Conceiving and mentally planning how to create or construct an object, figure, or representation; extend by physically designing through diagramming or mapping	Caldera et al., 1999; Sarama & Clements, 2009
diagramming	Creating drawn figures to map real space and structures to convey meanings about dynamic spatial structures in a way that supports informed interpreting	Davis et al., 2015; Hegarty & Just, 1993; Heiser & Tversky, 2002; NRC, 2006; Steenpaß & Steinbring, 2013; Stieff, 2007; Taylor & Hutton, 2013
modeling	Constructing (scale or dimension-shifted) representations of real spaces to simplify problems when interpreting information	Blades et al., 2004; NCTM, 2000; NRC, 2006; Sarama & Clements, 2009
relating	Showing or establishing connections between two or more [things] to make sense of how [things] are spatially organized	Davis et al., 2015; DeLoache, 1987; Kastens & Ishikawa, 2006; NRC, 2006; Newcombe & Sluzenski, 2004; Sarama & Clements, 2009; Vasilyeva & Huttonlocher, 2004; Uttal et al., 2013
symmetrizing	Interpreting and/or explaining balanced proportions through equivalent structures, often as a bilateral reflection	Bryant, 2008; Davis et al., 2015; Sarama & Clements, 2009; Sarama et al., 1996
Sensating	Perceiving through the senses	Davis et al., 2015
imagining	Forming (a) mental image(s); integrates with visualization because to visualize, one must form and hold mental images to conduct mental transformations	Davis et al., 2015; Hegarty & Waller, 2005; Rieser et al., 1994; Uttal et al., 2013
*perspective-taking**	Imagining or recognizing a location or object from another point of view	Muir & Cheek, 1986
projecting	Devising and presenting frameworks or diagrams	Davis et al., 2015; Sarama & Clements, 2009
propriocepting	Receiving stimuli via sensating regarding spatial awareness or movement	Newcombe & Huttonlocher, 2000; Sarama & Clements, 2009
tactilizing	Making perceptible by touch or tangible†	Davis et al., 2015
*visualizing**	Imagining and mentally transforming spatial representations	Arcavi, 2003; Chu & Kita, 2011; French et al., 1963; Hegarty & Waller, 2005; Mix & Cheng, 2012; NCTM, 2000; Presmeg, 1997

Note. * Only explicit definitions located in the sources were used to derive these terms' definitions. For all others, dictionary sources supported the adopted, adapted, and inferred definitions. †All skills can be enacted physically or mentally unless specifically noted.

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