Computational Thinking Workshop: A New Way to Learn and Teach Mathematics

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Abstract: In this digital era, technology has entered every aspect of our life, including educational system. Computational thinking (CT) and programming are a relatively recent part of certain school curricula. The idea of CT was originated in 1950s, and the first usage of the term CT was by Papert in 1980; the notion/concept was refreshed by Wing in 2006. CT is the focus of attention for many researchers, such as Gadanidis , Namukasa, Kotsopoulos, Curzon, diSessa, Farris, Sengupta and so on ; they argued that using CT tools, ideas and activities in mathematics pedagogies and curricula contributes to learning in creative and imaginative ways. In this paper, the ways that students interact with their peers during CT and mathematical thinking activities are investigated in the context of an instrumental case study of 10 elementary students. Observational, interview, and reflection data collected during two workshops were analyzed to determine the ways in which the activities impacted students' interacting and understanding. Students engaged in three CT activities: symmetry app, Scratch program, and Sphero robot. As a result, CT activities allow students to learn mathematical concepts better, when they are working with CT ideas and activities. This study was limited in its sampling as it only focused on primary grades 3 - 6 in a private school. For future studies, the researchers suggest conducting a study that will include public schools and involve tools for teaching mathematics concepts.

Keywords: *Computational thinking (CT); Mathematical thinking (MT); Reform mathematics education; Mathematics concepts.*

Introduction

Computational thinking (CT) is the focus of attention for many education researchers, and they argue that by incorporating creative thinking tools, ideas, and activities into teaching mathematics, the learning experience becomes more imaginative and innovative (Namukasa et al., 2017). Zakaria et al. (2010) observed that many students found mathematics difficult, uninteresting, and not relevant to their personal life experience when they are taught in the traditional way. According to researchers' personal experiences with teaching math, the difficulties in mathematics education seem to stem from both the nature of the mathematics subject and the teaching approach, in relation to how children learn.

The idea that computational thinking (CT) going to be a basic competence for learners in the future has been noted by Wing (2006). Aho (2012) expanded on this concept, stating that "we consider computational thinking (CT) to be the thought processes involved in formulating problems, so their solutions can be represented as computational steps and algorithms" (p. 832). Several researchers, including Curzon (2014), Farris and Sengupta (2014), Gadanidis et al. (2017), Namukasa et al. (2017), and Kotsopoulos et al. (2017), explored the integration of CT in classrooms and have realized that using CT activities, tools, and processes can lead more engaging in mathematics learning and more

productive. Gadanidis (2015) discovered a connection between mathematics and CT and observed that learners have ability to understand abstract and complex concepts when they are presented in meaningful and simulated situations.

This article explores the nature of student engagement during workshop activities designed to engage students in both computational thinking and mathematics thinking in the context of an instrumental case study of 10 elementary students. In order to study the ways in which students interact during CT and mathematical thinking activities, two workshops were offered to the students during school day. These workshops were based on the CT and mathematical thinking activities designed by Namukasa (2017) and Gadanidis (2017). The goal of these workshops was to explore the potential of incorporating CT into mathematics education as a means of making the subject more interesting, accessible, and less intimidating for students. The findings of Grover and Pea (2013) support the idea that integrating CT into mathematics instruction is a promising approach to improving students' engagement with the subject. By incorporating CT activities and processes into mathematics instruction, students can develop a deeper understanding of mathematical concepts and a more positive attitude toward the subject. This, in turn, can help to prepare students for the technological demands of the future and support their overall academic success.

Researchers are exploring the utilization of CT activities in mathematics education to benefit students' learning in mathematics, especially the abstract concepts (Gadanidis et al., 2017). In other side, there are many students find mathematics difficult and uninteresting (Zakaria, Chin & Daud, 2010). Although the wider computational thinking literature —such as Grover and Pea (2013) and Brennan and Resnick (2012) — argued that computational thinking could be a better technique for helping students to understand concepts. Kopcha et al. (2020) believed more research in school contexts is needed to consider the effect of engaging CT activities in learning process such as coding by using robots, which means use the concepts to be tangible.

Therefore, for the study, the research questions are: What is the nature of student engagement during computational and mathematical thinking activities? Specifically: In what ways do students act and interact during these activities? and What are the views and feedback of students after their engagement with these activities?

Theoretical Framework

The theoretical formulations of social constructivism (Vygotsky,1980), constructionism (Papert, 1980), and phases of using digital technology (Borba et al., 2016) have been chosen as the   framework for this study.

Papert (1980) maintained that learners of mathematics and programming, as apprentices, engage in learning these disciplines in a similar way to professionals in the field. Papert focused on engagement students with each other. Papert's previous work with CT has been extended by other researchers to notions of learning as both a social-cultural and political-critical endeavour. These researchers include Kafai et al. (2019), who spoke of computational participation, and Di Sessa (2018), who discussed computational literacy. Papert focused on the cognitive domain and

saw CT tools, artefacts, and materials as key in the construction of knowledge needed by students while learning the school curriculum.

In constructionism, learning is viewed as a process of creating and constructing knowledge through engaging in hands-on, creative activities that involve the use of technology. It is based on the idea of learning by making, where students engage in active, experiential learning and create meaningful products as a part of their learning process (Tedre & Denning, 2016). Constructionism is associated on hands-on experiences, knowledge construction principles, and active learning put forth by Bruner (2009), and it has been shown to be effective in promoting student engagement and learning outcomes (Kafai et al., 2019). This theoretical framework is particularly relevant to the integration of CT and mathematical thinking activities, as it highlights the importance of hands-on, meaningful, and socially interactive experiences in promoting student learning and engagement.

Bruner's (2009) theories of education emphasize the importance of creating opportunities for students to construct their own knowledge through experiences and interactions with the environment. He also emphasized that learning should be an active process and that students should be able to build on their prior experiences and understanding. These principles align with the ideas of constructionism, where learning is viewed as a process of building and reconstructing knowledge through hands-on and creative activities.

Similarly, Vygotsky's (1980) social constructivist theory highlights the role of social interaction and collaboration in the learning process. According to this theory, students learn best when they engage in activities and discussions with others, as this allows them to construct their own understanding based on the perspectives and knowledge of their peers, and also this theory emphasizes the importance of scaffolding, where teachers provide support to students to help them achieve their full potential (Vygotsky's, 1980).

These theories of learning provide a strong foundation for understanding the potential benefits of engaging in CT activities. By providing opportunities for students to actively construct and reconstruct their own understanding through hands-on activities, they are more likely to develop a deep and meaningful understanding of the concepts involved. Furthermore, the social and collaborative nature of these activities provides opportunities for students to engage in meaningful discussions and gain new insights from their peers.

Furthermore, Namukasa et al. (2017) noted that by allowing students to create their own projects, they can develop their own understanding of the concepts they are learning and form a deeper connection to the material. Also, Gadanidis et al. (2017) noted that CT activities, tools, and processes can play a significant role in enhancing mathematics learning experiences, and the use of CT tools can help students to visualize and interact with mathematical concepts, making them easier to understand and remember. As Bruner (2009) stated, during CT activities, students are presented with opportunities to perform learning as an effective process, and CT activities support learners to make mathematical concepts more masterly, when they are performing CT activities and ideas.

Literature Review

To situate this study, 1 reviewed the following: 1- Integration of CT activities in teaching mathematics, and 2- Reform in mathematics teaching and learning.

1- Integration of CT Activities in Teaching Mathematic

CT in K-8: Sanford and Naidu (2016), among other researchers, noted the affordances and benefits of CT to K-12 students and highlighted "the importance of adding 'computational thinking' as a core ability that every child must learn" (p. 23). Papert (1980), Kafai et al. (2019), and Gadanidis (2015), these researchers observed CT supports to the modification of teaching and learning approach. Angeli et al. (2016) and Higginson (2017) forecasted that is likely to continue, and more schools and educational institutions are expected to adopt CT curriculum into their curricula in the coming years. This is already the case in several European countries, including Finland and Sweden (Bocconi et al., 2016). In addition, Curzon et al. (2014), Gadanidis (2015) and Angeli et al. (2016) stated that, for example, many provinces in Canada are already integrating CT in their programmatic curricula. Other countries like Korea and the United Kingdom have also introduced computing syllabi to make CT a fundamental part in the curriculum.

Integration rather than a sole computer course for K-8: In some cases, like for grades 6–8 in British Columbia (2016) and K- 12 in the UK, CT has been introduced as a separate stand-alone curriculum. In this context, unfortunately, the opportunity to leverage the extended nature of learners' engagement in CT and gaming activities could easily be missed. Many curriculum studies researchers maintain that CT could be integrated in already existing curriculum disciplines. Farris and Sengupta (2014) argued that computational aspects are especially important in K-12 education, where students are exposed to a wide range of mathematical and scientific concepts. Lu and Fletcher (2009) declared that computational thinking (CT) should be considered as a fundamental skill, along with reading, writing, and mathematics. They suggest that CT should be integrated into the curriculum to assist students for improving their ability to think abstractly, solve complex problems, and design solutions that are effective, efficient, and scalable. Further, Sun and Zhou (2022) claimed that it does not matter when you begin learning programing. However, the time spent in learn programing has real effectiveness in learning. Thus, students who start learning programing in early years, they have high CT level (Sun & Zhou, 2022). By learning CT, students can develop a deeper understanding of how to use technology in a responsible and ethical way and be better prepared for the demands of the 21st-century workforce. By including CT as a fundamental skill, Lu and Fletcher (2009) aimed to provide a framework for how educators can prioritize the teaching of CT in K-12 education, and ensure that students are well-prepared for the future. According to Wang et al. (2021) and Ortiz et al. (2015), integration of science, technology, engineering and mathematics (STEM) benefits students to understand abstract concepts and engage them in with real world. Also, Fowler et al. (2021) recommended that teachers might work on extending their students' understandings of STEM skills. In addition, Barr et al. (2011) believed that educators can help students to prepare them for the challenges of the 21st-century workforce and give them the tools they need to

succeed in an increasingly technology-driven world. Also, using technology makes mathematics more valued (Duzhin & Tan, 2023).

Research on the nature of students' engagement during computational and curricular thinking is currently in its infancy, with researchers like Gadanidis (2017) exploring the affordances of CT tools and activities, Kafai exploring activities in after school programs, Aho (2012) exploring computational thinking (CT) in problem-solving contexts, and Wilkerson-Jerde et al. (2015) exploring that the design of computational modeling environments for modeling curricular concepts. Only a few of these studies, like those by Gadanidis, are carried out in Canada. Yet, four provinces – British Colombia, Nova Scotia, Alberta, and New Brunswick – have in the past 5 years integrated CT in their programmatic curricula.

Integration of CT in the mathematics K-8 curricular: Certain new curricula policy documents for mathematics, such as the Ontario K-8 Mathematics (OME, 2020), the K-12 Brazil Mathematics (BNCC, 2017) and the K-4 draft Alberta mathematics and science curricula and integrate CT in the curriculum of mathematics (Government of Canada, 2017). The integration of CT in mathematics education has been observed by several researchers, such as Gadanidis (2015) and Kafai et al. (2014), to fit with school mathematics instructional reform. For example, integrating CT reinforces problem solving through hands-on activities and real-world contexts. Kafai et al. (2014) sees incorporating making in teaching "as an unprecedented opportunity for educators to advance a progressive educational agenda in which project-based, interest-driven learning are the center stage of students' educational experiences" (p. 615).

2- Reform in Mathematics Teaching and Learning

Classroom environment: According to Haeck et al. (2011), establishing a classroom environment to encourage the improvement of mathematical reasoning through collaborative problem-solving is the main purpose of the reform. Furthure, Suurtamm et al. (2010) mentioned that aiding teachers with development of a classroom environment is major purpose of mathematics education reform, which could assist mathematical thinking through combined problem-solving. Thus, the goal is to reduce the opt-out rate from STEM fields and the dropout rate. By creating a classroom environment that emphasizes collaboration and problem-solving, Haeck et al. (2011) believed that students can develop a deeper understanding of mathematical concepts and improve their overall performance. Furthermore, this type of learning environment can help to engage students who might otherwise be uninterested in mathematics, and provide them with the foundation they need to pursue careers in STEM fields. Overall, the reform is aimed at improving student achievement and preparing students for the demands of the 21st-century workforce.

In addition, Kyriakides et al. (2016) stated that the pedagogical role of technology helps students to expand their understanding of mathematics. Recently, Vallera and Bodzin (2017) declared that the combination of technology and project-based learning provide students a rich and engaging learning experience that prepares them for the demands of the 21st-century workforce. Furthermore, Haeck et al. (2011) believed that schools can provide students with a rich and engaging learning experience that prepares them for the demands of the 21st-century workforce.

Furthermore, this type of learning environment can help to improve critical thinking skills, as well as encourage communication and collaboration, which are important skills for success in both school and the workplace.

The revised Ontario curriculum aims to provide students with a rich and engaging learning experience that prepares them for success in both school and the workplace. (OME, 2020). According to Ross et al. (2002), the classrooms must be formed in a way that encourages student-to-student interactions. This type of learning environment is constantly changing and dynamic not as a fixed environment.

Suurtamm et al. (2010) and Haeck et al. (2011) both emphasize the importance of facilitating student discussions involving mathematical reasoning. While it is important to help students learn procedures, processes, and concepts, it is also crucial to encourage the construction of new knowledge. However, finding the right balance between these two goals can be a challenging task for teachers. Suurtamm et al. (2010) suggest that teachers must be skilled in creating a learning environment that supports student reasoning and discussions, while also providing structure and guidance to ensure that students are able to make progress.

Traditional way: Haeck et al. (2011), Ross et al. (2002), and ICMI (2017) all emphasized the importance of moving away from traditional or academic approaches to teaching mathematics and towards real-world and a more comprehensive approach. This type of learning environment provides students opportunities to engage with open-ended and complex problems that are implanted in real-life contexts, and the same time employ mathematical tools and strategies to solve these problems.

The integration of CT in mathematics education has been shown to have a number of benefits. According to Barr & Stephenson (2011), the integration of CT can help to change the traditional way of teaching mathematics and contribute to the development of more dynamic and engaging learning environments. Angeli et al. (2016) suggest that CT framework concentrates on real-world problems, while Aho (2012) notes that integrating CT in mathematics teaching can lead to the development of critical thinking, complex and open-ended problems, and use of mathematical tools to support learning. Despite the spread of technological integration in education successfully, it still faces many challenges and unsolved issues until this moment. Likewise, the effectiveness of technological integration needs to be more understood for children's learning (Dorouka et al., 2020).

Overall, these authors suggest that the integration of CT has a main role in mathematics education reform by helping to create more dynamic and engaging learning environments that enable students to develop a deeper understanding of mathematical concepts and to apply these concepts to real-world problems.

Methodology

This instrumental case study research is to investigate the integration of CT activities in mathematics workshops for students. It relied on a qualitative research method as it gives a researcher more knowledge about the participants,

and allows researcher to obtain a richer knowledge of the study object and its density (Creswell, 2015). Case study research was most appropriate as it allowed me to gather the data, which related to the holistic and meaningful characteristics of real-life events, and to gather precious data by using different data collection approaches such as triangulation (Yin, 2009). In this study, observation, reflection forms, photos, interviews, audio records, and photocopies, in the workshop were analyzed.

Participants and sessions: Data for this study were collected from CT and MT workshops completed over one month during the spring of 2018. The participants were six students from grades three and four, and four students from grades five and six. All were joined in a private school located in an urban area of a city in Southwestern Ontario, Canada. Table 1 outlines the participants who attended the workshops. They had been invited to CT and MT activity workshops: these were separated into two sessions by grades: grades three and four, and grades five and six. The workshops were during school day hours. Each workshop session lasted 75 minutes (See Table 2). The student participants - except student 9 and student 10 -were also interviewed individually for approximately 15 to 20 minutes after participating in the workshops. Two classroom teachers and the parents or guardians of each student were present and interacted with their children during the workshops.

Table 1

Description of Participants in the Workshops

Students	Grades	First/Second session	Interviewed/ not
Boy 1	3	First	Interviewed
Boy 2 and Boy 3	4	First	Interviewed
Boy 4	6	Seconed	Interviewed
Girl 1	5	Seconed	Interviewed
Girl 2	3	First	Interviewed
Boy 5 and Boy 6	4	First	Interviewed
Boy 7 and Boy 8	5	Seconed	Not Interviewed

Note: The participants who attended the workshops.

Table 2

Outline of the Workshops

Content	Time	Activity
Day 1/ Grades 3&4	75 mins	Symmetry[1], Sphero & Scratch [2]
Day 2/ Grades 5&6	75 mins	Symmetry, Sphero & Scratch

Note: The two sessions details.

Data analysis: Data from the researcher's observations, feedback from participants, photo images, and interview transcripts were organized for analysis. The analysis followed the work of Cohen, Manion, and Morrison (2007).

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[1] http://researchideas.ca/sym/s2/

[2] https://scratch.mit.edu/projects/editor/?tip_bar=home#editor

First, the data were organized by individual child participant, where each participant's responses were presented in a separate file to maintain the distinctly among participants' responses. This enabled the in-case (i.e., for individual participants) analysis, which was then followed by cross-case (i.e., comparing participants) analysis. The data were analyzed manually and then the researcher created the codes depending on research questions and the exist data. This involved clustering text and image data into different codes. Our focus was on the nature of engagement, specifically their ways of acting and interacting during, and their views and feedback after, engagement in computational and mathematics thinking activities.

The activities: All children that participated in the workshops worked on three CT activities: a website-based app for coding visual designs – Scratch, a website-based CT app for exploring math concepts –Symmetry, and a robot coded on tablets or phones – Sphero. The Scratch activity involved students designing objects using geometric and numerical properties as well as other mathematics topics. The Symmetry app involved both coding and mathematics. Students used it to assemble code that resulted in transformations like the rotation and reflection of geometric shapes. The Sphero robot activities involved assembling code on an iPad app to execute making Sphero robot moves and color actions with the robot based on numerical, geometric, and patterning ideas. This is to clarify mathematics concepts. The researcher provided task instructions for coding screen characters, for visualizations, and for coding the robots, with a goal to simulate mathematics concepts using with CT tools and ideas.

The activities we selected involved visual programming similar to Logo programming settings designed for learners (Papert, 1980); which this is the constructionism theory of learning. They were established on the idea of learning by making and the learning process as one of reconstruction (Tedre & Denning, 2016). We structured the learning to include social environments in which the parents and students interacted during the students' activities. Each of the activities was based on the three educative skills of CT activities: "tinkering," "making," and "remixing" recognized by Kotsopoulos et al. (2017). Students were not engaged in 'unplugged' (without computers) experiences but rather they engaged in tinkering experiences requiring engagement with coding and adjusting the code based on the feedback from the characters, mathematics shapes, and robots coded. Students involved in digital making and remixing the sample code provided in order to create new mathematics shapes, paths, and transformations.

Results and Dissction

The results reported the nature of the engagement of middle elementary students during CT and MT activities and are grouped under two main themes:1- Learning of mathematics and CT; 2- The Nature of CT Activities.

Learning of Mathematics and CT

In this section, we present our findings under two sub-themes: learning mathematics topics through CT (including coding and CT), and a new way to learn and teach mathematics. The following is a summary of finding's table which includes students' feedback on computational thinking workshop:

Table 3

Participants' Views and Feedback on the CT Activities

Students	Grades	Students' Quotes of CT Workshop	CT Enrich Math
Boy 1	3	"Fun learning, so the good way"	Agreed
Boy 2	4	"Amazing I love it"	Agreed
Boy 3	4	"It was interesting, I was excited	Agreed
Boy 4	6	"I find the workshop fun and helpful"	Agreed
Girl 1	5	"I really like it"	Agreed
Girl 2	3	"Very interesting"	Agreed
Boy 5	4	"It is fun"	Agreed
Boy 6	4	"I find it very fun and teaches me more"	Agreed
Boy 7	5	"like it"	Agreed
Boy 8	5	"Tt's really fun"	Agreed

Note: CT activities enriched students' understanding of math concepts employed within the activities.

Regarding the previous table, the next section is explaining the two themes that were extracted:

Learning mathematics topics through CT: Most of the students spoke about learning mathematics topics through the CT activities offered during the workshops. They thought that these activities helped them to explore specific mathematics topics like geometry, transformations, patterns, and angles. For example, Boy 1 said, "I learned that when I made a triangle, I had to put 120 degrees to make it work for the exterior angle." Boy 3 said, "we learned geometry," and "length, time, angles, patterns…." Boy 4 said, "I liked this activity because I was surprised that there was symmetry [code in the] shapes, we can do it easily." Girl 2 said, "I learned that if you rotate the shape, the numbers [labelling the vertices] are different, not the shape." "It helped me make sense about the angles." Boy 4 said that he learned how to calculate speed and time: "go forward at 50% for 3 seconds and change color to green."

Figures 1 and 2 show how students created perception of the activities in the Symmetry app, which focuses on geometry and transformation. All students discussed about the code and mathematics concepts, which they learned. The researcher noted that learners were highly involved in CT and MT activities when working the Symmetry app. The students' experiences and attitudes in relation to the activities appeared positive, as noted in the researcher's observation notes.

The values entered in the code on the right results in a rotation of the image on the left (see Figure 2). The numbers and dots – 1, 2, 3, 4 are used to label the vertices, so students see the change resulting from the rotation. For example, the rotation by 90 degrees repeated 10 times results in unique rotations, at 90, 180, 270 and 360 degrees, also shown by 4 unique combinations of numbers.

Coding + CT: Most of the students commented that they learned how to code and play coding games. For example, Boy 1 said that he learned "how to make code" and that "you can make it [the Sphero robot] dance, you can play games, etc." Girl 1 also noticed that "it can move using a code," and Boy 6 was surprised by "how the code worked

and how fast it [the robot] went and all the codes it had." As seen in Figures 3 and 4, students assembled code on the devices to make shapes by using the Sphero robot.
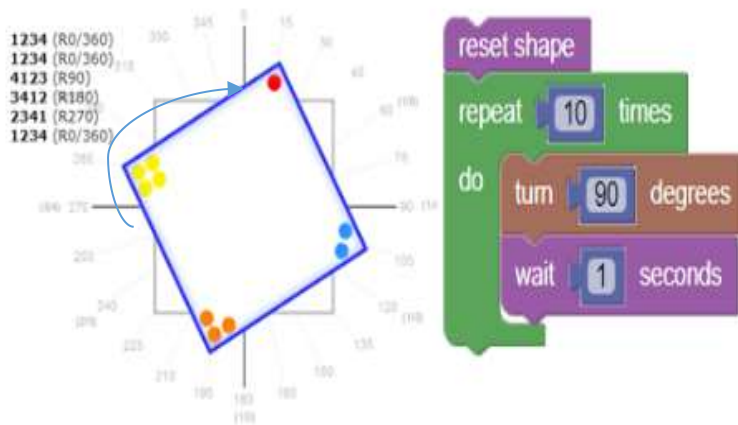
Figure 1

Symmetry App Activity During Workshop[3]
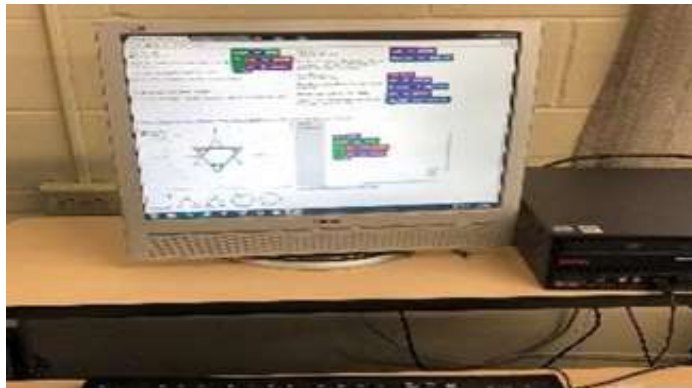
Note 1: Students working on symmetry app.

Figure 2[4]

An Image of Rotating a Square Shape in the Symmetry Activity[5]

Note: More clarifiction photo.

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[3] Two children exploring the rotation of a triangle in the Symmetry activity while taking notes on a page in the handout
[4] Figure 2 is for more clarification than Figure 1
[5] http://mathsurprise.ca/apps/sym/rotation-reflection/

Figure 3

Children Participating in the Sphero Robot Activity [6]

Note: Students working on Sphero robot.

Figure 4

Creating a Code of Sphero Robot to Make a Square[7]

Note: More clarifiction photo.

A new way to learn and teach mathematics: CT activities are a different and new approach to understand mathematics as most participants said. For example, initially Boy 5 found the CT activities "confusing because it's new stuff." However, "once the children realize what are supposed to do in CT activities, they will be understandable." Afterward, Boy 5 thought that CT activities were "very useful" and said that he was "very impressed."

Discussion on Learning Mathematics and CT: This theme is a collection of all participants' viewpoints on how CT activities boost teaching and learning mathematics. It displays the nature of integration of students learning in CT activities. Furthermore, this theme includes the context of the use mathematics in daily life, and also the place of CT activities on geometry, coding and games, and benefits and challenges of CT workshops.

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[6] Students worked in small groups to code the Sphero robot to perform actions by assembling and executing code on a tablet

[7] The code blocks assembled by the children by dragging the commands involved motion blocks for moving and turning by specified time, degrees and speed, as well as control ideas of repeating and choosing the time between commands

Our findings on learning mathematics topics through CT, involving coding screen characters and robots, are consistent with the literature on CT in general, and CT in mathematics in particular. Block-based programing software was introduced in the past decade to make learning computer science programing more available for learners (Kafai, 2016). Resnick et al. (2009) stated that Scratch, which is derived from Logo programming, is one the most common CT apps used by children. Borba et al. (2016) observed that coding activities, which involve dragging, such as Scratch, increase students' abilities to experiment with mathematics. Further, the Scratch program gives children opportunities to learn mathematics concepts creatively and collaboratively (Resnick et al., 2009). Resnick et al. further argued that Scratch is suitable for many different ages and allows students to create, share, and remix projects, which include games, stories and simulations. Thus, Benton et al. (2017) highly recommended including CT activities, especially the Scratch program, which integrates a lot of mathematics ideas such as geometry in the mathematics curricula.

According to Kopcha et al. (2020), "educational robotics activities are one way to support children's development of computational thinking skills and promote computer science" (p. 6). They added that activities with robotics allow students to divide a larger task into numerical steps or commands, and thus to understand coding. They also build students' skills in finding alternative solutions, by applying mathematical concepts (Kopcha et al., 2020; Zuod, 2019). Kopcha et al. added that "recent evidence suggests that children draw on their bodily understanding of the world when engaging in mathematics (e.g., Ferrara, 2014) as well as in educational robotics (Sung et al., 2017)" (p. 6). Perhaps this is what students were referring to when they talked about learning mathematics in new ways.

In addition, Merino-Armero et al. (2018) claimed that students are paying more attention when they are using robots during their learning and improving their learning. Also, Merino-Armero et al. (2018) stated "we can also conclude that the inclusion of CT with educational robotics is highly motivating in the short term." (p. 197). Thus, researchers such as Zhong and Xia (2020) hope that the research in the future will focus on robot-assisted mathematics education and conduct more serious research to explore teaching and learning mathematics with robotics assistance. As shown by the students' comments about the mathematics topics they encountered during the CT activities, these activities have the potential to create additional understanding in learning mathematics in a new way by using coding. This confirms what Lee et al. (2011) noticed: the importantly of establishing a stimulant learning environment and conduct more CT research, this leads to support the growth of students' CT skills.

Student comments on coding and CT, especially what had surprised them and what they had learned. Namukasa et al. (2017) observed that the learners can comprehend complex and abstract mathematical concepts through activities that use CT tools, robots, coding apps, and games. Researchers such as Namukasa et al. (2017), Gadanidis et al. (2017), Farris and Sengupta (2014), Kotsopoulos et al. (2017), and Curzon (2014), examined the integration of CT and MT in K-8 classrooms, and these researchers noticed that CT tools, activities, and processes have potential to make advanced mathematics learning experiences more productive, more interesting, and easier, for students at all learning levels.

The finding of students which they are stated the new way in learning mathematics through CT, and this is in line with Gadanidis (2015), who noticed that CT activities assisted changes in the traditional methods in mathematics education. Also, this finding showed what Lu and Fletcher (2009) founded: teaching CT is an essential skill that should be utilized as a fundamental one for students, beside teaching writing, reading and mathematics.

Coding was useful since it involved exploring mathematical concepts through playing with robots and apps. In addition, CT activities aided learners the way that learn how to code digital tangibles, screen characters, and mathematics objects. This, in turn, made it easier for them to understand abstract mathematical concepts, including angle measurements.

The Nature of CT Activities

Participant experiences of the activities: All participants commented on the difficulty that they experienced during the CT activities. In general, the robot activity was considered by students that it was reasonably easy, and students expressed that they enjoyed the robot activity the most. All students found the Sphero easier than other activities because it included physical activity that let the students get up and move around. The other activities included a web-based app for manipulating mathematics objects (the Symmetry app) and software (Scratch) for designing objects using mathematics concepts.

Boys 2, 4, and 6 commented that they found the Symmetry app, in which they assembled code for transformations of geometrical shapes, difficult and they did not enjoy it much. Their explanation was that this activity is similar to mathematics as it is currently taught in classrooms these views of the Symmetry app, which is designed for exploring mathematics topics, may imply that certain children preferred the out-of-the classroom workshop activities which were significantly different from the in-classroom mathematics activities that they currently encounter i.e., those specified in their mathematics curriculum, which use resources such as textbooks, notebooks, work sheets, and on the computers.

Boys 1, 3, and 5 said they did not enjoy the Scratch program, which involved designing objects using mathematics topics. They each explained that they found it "hard." This reaction to the Scratch program can be explained by the fact that the Scratch environment has several design features that students may not grasp in a single workshop session. Students explored Scratch as the last activity of the workshops, and there was not enough time to complete and consolidate their learning in this activity.

Overall, most participants, found the Sphero robot activities, in which they coded a ball-shaped robot to move in a path related to its geometrical properties, the most engaging. They said it granted them the ability to play, move, touch, and learn at the same time through simulations of curricular concepts in what felt like the real world of physical objects and processes like motion. This highlights the need to focus on the ways in which to combine or sequence the activities in order to provide the maximum benefit for learning, comfort and efficacy for the learners.

Participants' views and feedback on the activities: Students' views, noted in the researchers' notes, interview transcripts, and student feedback forms, commonly referenced their positive experiences when working with the CT and MT activities. Most of the participants commented on how much they enjoyed the workshops. Even during the beginning of the workshops, where some participants found the activities to be new and difficult to engage with, participants' views were mostly positive. For example, Boy 1 said it was "fun learning, so the good way." Boy 2 said the workshop session was "amazing I love it," and Boy 3 said "it was interesting, I was excited." Boy 4 said "I find the workshop fun and helpful." Girl 1 stated "I really like it." Boy 5 said "it is fun," and Boy 6 said, "I find it very fun." All students said they liked the CT activities in the workshop, and most said that they were glad they had participated.

Participants' suggestions on the design of the activities: Students contributed several suggestions to improve the design of the CT activities and the workshops in general. Overall, the most common suggestions were extending the time for the workshop, adding variety by doing more activities, and including more clarity in the instructions. Some thought it would be best to participate in more activities and spend more time in the workshops, especially with their favorite activities.

The benefits of the students' engagement in CT activities from the participants' self-reported data appeared to outweigh, in terms of the number of benefits listed by each student. Some of the initial difficulty in understanding the new context of the CT tools. A few students commented on this, and the researcher observed that at the beginning of both workshops the participants were grappling with the instructions for coding robots, screen characters, and visualizations to simulate mathematics concepts.

Discussion on experiences, views, feedback and suggestions: Our findings on the nature of CT activities involved in coding screen characters and robots is in line with the literature on CT in mathematics learning contexts. Computational thinking activities are used increasingly in teaching mathematics curricula, and researchers such as Gadanidis et al. (2017) and Namukasa et al. (2017) have noted that there are many computational tools and activities that can be used in mathematics teaching and learning. They have explored the teaching practices for using the tools and materials when integrating mathematics and CT concepts in elementary classrooms. Hsu et al. (2018) concluded that "to help students correctly understand and integrate into the information society, it is not enough to cultivate their creative ability and improve their digital literacy; they also need to enhance their CT capability, learn to utilize new technological skills, and take full advantage of such skills to adjust to the rapid change in the information society" (p. 308).

The finding that children preferred CT activities that were set in the realm of physical objects and motion is similar to the results of the study conducted by Resnick (1995), who observed that CT is about "how they [students] think about and make sense of the world" (p. 31).

The comments of students reflecting mostly positive views of the workshops confirm the findings of Vallera and Bodzin (2017), who suggest that combining technology with authentic project-based learning challenges using real-world examples can help children develop a better grasp of complex and abstract concepts. This finding supports the premise of incorporating CT activities in teaching mathematics and in the mathematics curriculum. These comments support the conclusion of Wing (2006) that "computational thinking will be a fundamental skill that is used by everyone in the world in the 21st century" (p. 2). The comments also validate the recommendations of Sanford and Naidu (2016) that CT activities, which are more recent learning activities not afforded to adults, should be offered to parents as well.

Conclusion and Limitations

Educators and researchers recommended continuously using CT activities and tools in mathematics education, and they consider that CT activities and tools lead to improved students' understanding, achievement, and enjoyment in learning process. Also, researchers expressed about the needs of new research on how the successfully integrate CT activities and tools into the teaching curriculum content.

This study was limited in its sampling in the following ways: it focused only on primary grades 3 – 6, it was carried out in a private school, the time and context of the workshops were during school hours, with the researchers (not the teachers) teaching the workshop. Given the context of the sessions, the researchers only engaged students in introductory CT activities. For future studies, the researcher suggested conducting a study that will include public schools and will involve more specific CT tools that extend CT tasks for teaching mathematics concepts. The researcher also recommended conducting CT workshops over the course of a three-day period so that children can participate in each activity with ample time each day rather than completing three distinct activities in one short session.

Nonetheless, the findings of this study have implications for selecting CT tools and for designing CT activities for CT and mathematics workshops. Students appeared to enjoy activities that involved more real-world (specifically physical material world, with robots) simulations of mathematics concepts more than the other simulations; yet these other simulations were more closely matched with the mathematics content specified in the curriculum and taught by their teachers in the classroom. Also, the simulations embodied the potential to explore other mathematics topics such as geometry. This finding raises questions for further studies on the ways in which to combine or sequence the activities to benefit both cognitive (e.g., learning) and affective (e.g., enjoyment and efficacy) for the learners. Our study has implications for practice, particularly for out-of-the-classroom CT workshops to include extended explorations over several sessions. Future studies may explore these extended contexts as well as contexts which

study teacher and parent participation in using the digital and information technologies, already present in classrooms and in homes, for teaching mathematics lessons.

Integrating CT and existing curriculum content is a promising way to utilize the digital and information technologies, and they already present in classrooms and in homes for teaching mathematics lessons. This new approach will address the world-wide challenge (OECD, 2020) of a lack of the extensive use of these technologies as technologies-to-think-with (Borba & Villarreal, 2005) when learners are exploring mathematics. CT activities allow students to learn mathematical concepts when they are playing or working with CT ideas and activities.

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