Journal of Research in Science, Mathematics and Technology Education

https://jrsmte.com/

An APOS Analysis of Undergraduate Mathematics Students' Understanding of the Limit of a Function Concept: A Case of a University in Zimbabwe

Edmore Mangwende

University of Zimbabwe, Zimbabwe

Aneshkumar Maharaj

University of KwaZulu-Natal, South Africa

Abstract: Despite the foundational importance of limits in calculus, many students struggle to move beyond procedural knowledge. This study analyzed fifty-one first-year mathematics students using the APOS-ACE framework to identify specific cognitive barriers. Results show that after standard instruction, more than half of the students could not reach the "Object" level of understanding. These findings suggest that traditional lecture formats may be insufficient; instead, the researchers advocate for a more dynamic instructional approach incorporating peer collaboration and diverse visual activities to facilitate the construction of complex mental schemas.
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Keywords: APOS-ACE; limit of a function; mathematics student; mental structure; undergraduate mathematics.
DOI: https://doi.org/10.31756/jrsmte.931

Introduction

The concept of the "limit of a function" serves as the theoretical cornerstone of calculus, providing the formal basis for derivatives, integrals, convergence, and continuity. Mastery of limits is a prerequisite for navigating the rigorous analysis of real-valued functions. In the Zimbabwean higher education context, undergraduate mathematics students are introduced to this concept during their first year within the compulsory Calculus 1 module. This course spans foundational topics including sequences, series, and number systems and extends to critical theorems such as the Mean Value Theorem, Cauchy's convergence criterion, and the Fundamental Theorem of Calculus. Because these advanced constructs are defined through the lens of limits, a student's success in the broader mathematical curriculum is often contingent upon their grasp of this single concept.

Despite its importance, pedagogical observation suggests that many students struggle to move beyond a procedural understanding of Calculus 1, largely due to deep-seated difficulties with the limit concept itself. This study seeks to bridge the gap between curriculum requirements and student performance by exploring the specific levels of understanding and common misconceptions held by undergraduates. By identifying where conceptual breakdowns occur, this research aims to provide university-level educators with the insights necessary to design targeted learning activities that address and rectify these cognitive hurdles.

Research Questions

- What levels of understanding do mathematics students exhibit regarding the limits of functions?
- What specific misconceptions do students hold when engaging with the limit concept?

Theoretical Framework

This study utilized the APOS theory of conceptual development. APOS, an acronym for Action, Process, Object, and Schema, describes the successive stages of mental structures through which students progress to internalize mathematical concepts. Grounded in Piaget's genetic epistemology, the theory suggests these stages do not necessarily occur in a linear progression (Oktac, Trigueros, & Romo, 2019). Initially advocated by Dubinsky (1984) and further refined by Asiala et al. (1996), the framework serves as both a pedagogical tool and a research methodology. Weller, Arnon, and Dubinsky (2009) characterize APOS theory as a lens through which educators can analyze how learners construct mathematical knowledge via the following four components: action, process, object and schema.

Action

An action is a physical or mental reaction to external stimuli. It is analogous to a person assembling a gadget by strictly following a manual. In mathematics learning, these stimuli may include demonstrations or instructions from textbooks, peers, or teachers. A student at this stage of mental development performs tasks by imitating a demonstration or following a prescribed set of steps without necessarily understanding the underlying mechanics (Maharaj, 2010).

At the Action level, a student evaluates the limit of a function at a given point primarily through direct substitution. Understanding is considered limited to this stage if the student relies solely on substitution without understanding the underlying behavior, fails to apply substitution correctly and attempts to use substitution in cases where it is mathematically inapplicable.

Process

Through repeated reflection on an action, a student interiorizes it into a mental process. At this stage, the student becomes conscious of the operation and can perform it internally without external guidance. Unlike an action, a process is under the student's internal control (Mukuka & Tatira, 2024). According to Maharaj (2010), a process performs the same operations as an action but resides entirely in the mind. A student with a process conception can reflect upon, describe, reverse, or combine processes, imagining the sequence without needing to perform it physically (Asiala et al., 1996).

In the context of limits of functions, a student reaches the Process level once they have interiorized the actions of limit calculation into mental operations. At this stage, the student can analyze the behavior of $f(x)$ as x approaches a to determine the limit L, distinguish between the value of the function $f(a)$ and the limit of $f(x)$ at a and evaluate and compare right- and left-hand limits to determine if a unique limit exists.

Object

When a student fully grasps the processes and actions involved in a mathematical concept, they encapsulate the process into an object. The student then perceives the concept as a discrete entity upon which new actions and processes can act. A student at the object level can solve complex problems by coordinating multiple mental structures and can de-encapsulate the object back into its constituent processes when necessary. However, reaching the object conception remains a significant challenge for many mathematics students (Asiala et al., 1996).

In the case of a limit of a function, an Object understanding is achieved when the student views the limit process as a totality, a "thing" upon which new transformations can be performed. The student can combine various mental processes and actions fluidly and deconstruct the limit "object" to determine which formal methods (such as L' Hospital's Rule) are appropriate based on the function's nature.

Schema

A schema develops as a student interconnects actions, processes, and objects into a coherent cognitive structure (Trigueros et al., 2024). It serves as the framework the student uses to decide how to approach and solve a mathematical problem. When constructing higher-level schemas, a student may thematize an existing schema into an object. For example, when learning derivatives, a student might thematize their "limit" schema to treat it as a single object within the new context.

Student with fully developed Limit Schema have successfully interconnected actions, processes, and objects. This integrated framework allows them to solve complex, non-routine limit problems regardless of their form and determine and execute correct operations for limits involving sums, differences, products, and trigonometric functions.

Genetic Decomposition

When using APOS theory to analyze how students construct knowledge, researchers utilize a tool called genetic decomposition. This describes the specific behaviors students exhibit as they develop mental structures related to a concept. According to Cetin (2009), a genetic decomposition is theoretically constructed based on three factors: (1) The researcher's own understanding of the concept, (2) the historical development of the mathematical concept and (3) conclusions drawn from existing literature.

Genetic decomposition allows researchers to collect qualitative data on a student's level of understanding, typically after subjecting them to designed instruction and follow-up interviews. For the current study, the researchers analyzed students' written and oral responses against a genetic decomposition adapted from Maharaj (2010) and modified to fit the specific context of this study.

A common instructional design associated with APOS theory is the ACE teaching-learning cycle (Arnon et al., 2014). This pedagogical approach consists of three components: Activities (A): Students perform individual or small-group tasks in class to initiate the development of mental structures, Classroom discussions(C): Students share ideas and cross-examine their work and Exercises (E): The teacher facilitates by providing definitions, explanations, and clarity (Weller et al., 2011).

Previous studies on students' understanding of the limit of function concept

The Challenge of the Limit Concept

Despite its foundational importance in mathematics, the concept of a limit remains one of the most significant hurdles for undergraduate students. Research indicates that these challenges stem from several factors, including an inadequate grasp of the axioms of real numbers (Liang, 2016), difficulties in algebraic manipulation (Sebsibe et al., 2019), and a fundamental failure to decode the notations and symbols such as δ, ϵ and ∞ associated with the concept (Fernandez, 2004).

Genetic Decomposition and Formalization of a Limit of a Function

Cottrill et al. (1996) proposed a seven-step genetic decomposition to model the mental constructions required to understand the formal definition of a limit. Steps 1 to 4 focus on the transition from Action to Process. Students begin by evaluating $f(x)$ at a single point a, then move to a series of points near a, eventually interiorizing these actions into two coordinated processes: x approaching a, and $f(x)$ approaching L. Steps 5 to 7 focus on the formalization of these processes via the epsilon-delta definition:

$$\lim_{x\to a} f(x) = L \Leftrightarrow \forall\epsilon > 0, \exists\delta > 0{:}\, 0 < |x-a| < \delta \Longrightarrow |f(x) - L| < \epsilon \qquad (1)$$

While Cottrill et al. noted that students struggle significantly with this formalization, Swinyard and Larsen (2012) argued that the original genetic decomposition lacked sufficient empirical evidence to categorize the specific nature of these challenges.

Informal vs. Formal Concept Images

Fernandez (2004) identified a disconnection between a student's informal concept image (their intuitive understanding) and the formal definition. She observed that students often fail to understand what epsilon and delta represent, leading to a reliance on spontaneous conceptions rather than formal logic. Consequently, she argued that students should not be forced into formal notation prematurely. This is supported by Juter (2005), who noted that students frequently memorize the definition as a rote string of symbols without understanding its operational utility. Swinyard and Larsen (2012) further discovered that students are often reluctant to use the formal definition, preferring an "x-first" approach (focusing on x approaching a) rather than the "epsilon-first" approach required by the formal definition. They suggested that the mental process of finding a limit candidate is distinct from the process of verifying it, requiring a refinement of Cottrill's original steps.

Identifying the Research Gap

Recent studies, such as Ndagijimana et al. (2024), have categorized student errors into conceptual (e.g., failing to determine continuity) and procedural (e.g., using incorrect formulae). However, a critical gap remains in the existing body of research. While previous studies including Maharaj (2010) have utilized constructed genetic decompositions to theorize why students lack certain mental structures, there is a scarcity of empirical evidence derived directly from student work to support these claims.

Many existing studies focus on how students perceive the definition rather than how they apply it in problem-solving. This study addresses this gap by analyzing both the written responses and oral explanations of students. By evaluating these primary artifacts, we aim to confirm the existence (or absence) of the mental structures proposed in theoretical frameworks and provide a more granular view of how students operationalize the limit concept.

Methods

Research Design

This study adopted a qualitative design and was descriptive in nature. It utilized a case study design, focusing on first-year undergraduate mathematics students at a university in Zimbabwe.

Participants

The participants consisted of 51 first-year undergraduate mathematics students enrolled in Calculus 1, a compulsory first-semester course. This cohort represented the entire population of first-year mathematics students at the university during the first semester of the 2022 academic year. To ensure anonymity, the field researcher randomly assigned each participant a unique identifier ranging from 1 to 51.

Data Collection Procedure

Data were collected through a written test and semi-structured, face-to-face interviews. The field researcher interviewed students who displayed incorrect working or misconceptions in their test responses. The students were asked to justify their steps and explain their reasoning.

Pedagogical Framework: The ACE Cycle

The field researcher implemented the ACE (Activities, Class discussion, and Exercises) cycle as the primary instructional approach. During the tutorials, students collaborated in groups of five to complete activities focused on the limits of functions. Following these tasks, the instructor facilitated feedback through class-wide discussions. To conclude the session, students were assigned individual exercises to complete independently at home.

This model was selected because it facilitates a blend of collaborative learning and individual practice under the instructor's guidance. Given the condensed course schedule consisting of only 40 contact hours, the ACE cycle effectively managed the limited time by integrating structured homework. The course was delivered via two-hour lectures, held five times a week over a four-week period.

Lecture Structure and Time Allocation

The concept of "limits" was covered over three lecture periods. Each two-hour lecture was divided into distinct sessions: Session 1 (30 mins): Students performed individual tasks, Session 2 (30 mins): Students engaged in small-group discussions (groups of five) to review their individual work, Feedback and Instruction: Group representatives presented findings to the whole class, followed by the tutor clarifying critical concepts and Homework: At the end of each lecture, students were assigned exercises to complete either individually or in groups.

Instructional Progression

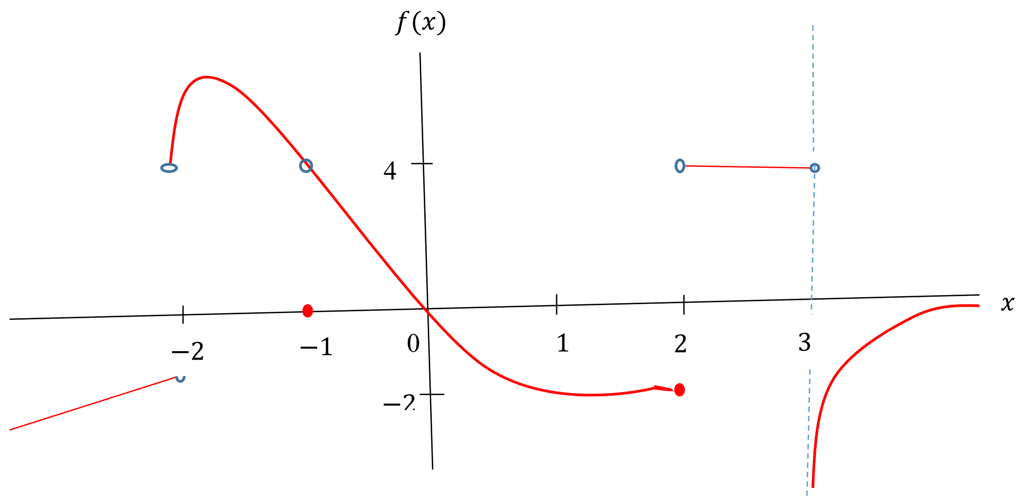
The curriculum progressed through the following stages: (1) Initial lecture: Focused on limits of functions in the form $\mathrm{f(x) = ax + b}$ and $\mathrm{f(x)} = \begin{cases} \mathrm{ax+b} & \mathrm{x \leq e} \\ \mathrm{cx+d} & \mathrm{x \geq e} \end{cases}$ (where $\mathrm{a, b, c, d}$ and e were constants). (2) Subsequent Lectures**:** Advanced to composite functions, trigonometric functions, and problems involving the notion of infinity.
At the conclusion of the third lecture, students completed a 30-minute summative test. The test items were as follows:

Question 1

Figure 1 shows the graph of the function, f(x)**.** Determine the limit of f(x) at the points where x is $-2, -1, 0, 2$ and 3**.**

Figure 1

Graph of the function f(x)



Question 2

a) Evaluate $\lim_{x \to 0} \frac{x}{x^3}$

b) Find $\lim_{x \to \infty} \text{Cos}\left(\frac{1}{x}\right)$

c) Given $f(x) = \begin{cases} x - 3 & x \leq 2 \\ 2 + x & x \geq 2 \end{cases}$ Find (i) $\lim_{x \to 2^+} f(x)$ (ii) $\lim_{x \to 2} f(x)$

To answer the first question correctly, researchers expected students to identify the specific characteristics of the function $f(x)$ provided in Table 1.

Table 1

Summary of the characteristics of $f(x)$ at the given points

Point on the x-axis	State of continuity, right-hand limit and left-hand limit of the function at the point	Conclusion on the limit at the point
$x = -2$	The function is not continuous. The left-hand limit is not equal to the right-hand limit.	Non existent
$x = -1$	The function is not continuous. The left-hand and the right-limit are equal.	4
$x = 0$	The function is continuous. The left-hand limit and the right-limit are equal.	0
$x = 2$	The function is not continuous. The left-hand limit is not equal to the right-hand limit.	Non existent
$x = 3$	The function is not continuous. The left-hand limit is not equal to the right-hand limit.	Non existent

The primary focus of this assessment was to test the students' understanding of the relationship between limits and continuity. Specifically, the researchers highlighted that: The existence of a limit at a point is a necessary but not sufficient condition for continuity. Conversely, a function can be defined at a point (continuous in a broad sense) while the limit fails to exist, or the limit can exist while the function remains discontinuous at that specific point. The second question tested proficiency in identifying and executing the necessary procedures to evaluate limits at the points provided.

Results

Table 2 presents the frequency and percentage of correct responses for the first item of the test.

Table 2

Number of correct responses for each item of the test $(n = 51)$

Point on the x-axis	$\mathbf{x = -2}$	$\mathbf{x = -1}$	$\mathbf{x = 0}$	$\mathbf{x = 2}$	$\mathbf{x = 3}$
Number of correct responses	21	33	49	17	15
Percentage of correct responses	41.1	64.7	96.1	33.3	29.4

An analysis of the results shows that cases where the limit of a function was equal to its value at a specific point, specifically where the function was continuous, were easy for most students. At the point [$x = 0$], 96.1% of students correctly identified the limit. According to our genetic decomposition, these students have attained at least an Action understanding of the limit concept. Conversely, the remaining 3.9% failed to identify the limit at this point and are considered not to have attained an Action level of understanding.

In the context of our genetic decomposition, students who recognized the non-existence of limits at points [$x = -2$], [$x = 2$], and [$x = 3$] demonstrated at least a Process understanding. This level requires students to interiorize and imagine the actions involved in finding a limit to conclude that it does not exist, specifically by realizing that the left-hand and right-hand limits are unequal. However, only 29.4% of students successfully navigated this at point [x=3].

Interviews with students who provided incorrect answers for Question 1 revealed several key misconceptions regarding the limit of a function. Table 3 gives the identified misconceptions.

Table 3

Identified misconceptions

Misconception	**Student Evidence**
Discontinuity implies a limit of zero	"At the point where [$x = 2$], the small circles are not shaded... the function is not continuous so the limit is zero." (Student 37)
Non-existence equals zero	"It is zero because it does not exist." (Student 3)
Confusing the limit with the function value, $\boldsymbol{f(a)}$	"The circle at [$x = -1$] is shaded meaning [$f(-1) = 0$]. Therefore the limit is [0]." (Student 12)
Zero as "No Limit"	"The value of the function at the point zero is zero, so there is no limit there." (Student 19)
Discontinuity implies non-existence	"The function shown is not continuous at [$x = -1$]. The limit does not exist." (Student 33)

Table 4 shows the numbers and percentages of correct responses for the second question of the test.

Table 4

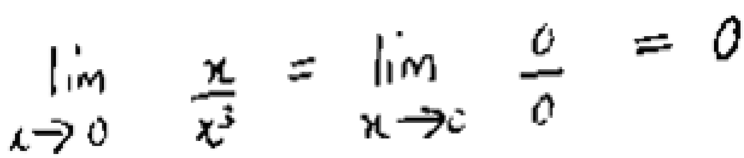
Number of correct responses per test item for question 2 $(n = 51)$

Question item	a	b	c(i)	c(ii)
Number of correct responses	19	10	50	37
Percentage of correct responses	37.3	19.6	98	72.5

The limit of the function in Question 2 could not be determined through direct substitution. Students who have not yet reached a process-level understanding of limits likely struggled with this task, as it requires analytical steps beyond rote computation. Despite this, over half of the participants attempted direct substitution, resulting in an indeterminate form. Student 27 (Figure 2) exemplifies this common struggle.

Figure 2

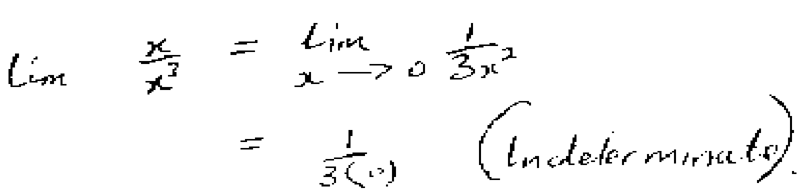
Response to question $2(a)$ *by Student 27*



Analysis of the student's work indicates that Student 27's comprehension of the limit concept is confined to an Action level of understanding. In contrast, Student 51 exhibited signs of a developing Process stage by recognizing the applicability of L' Hospital's rule; however, the subsequent execution suggests this understanding remains incomplete. This transition is evidenced in the work shown in Figure 3.

Figure 3

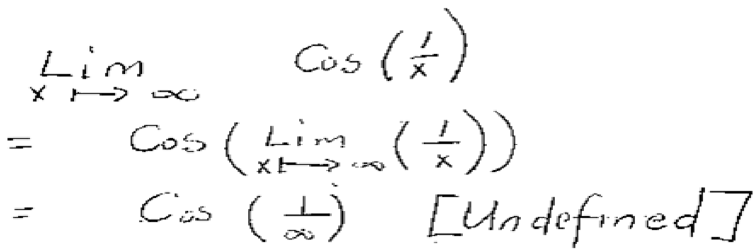
Response to question $2(a)$ *by Student 51*



Question 2(b) required students to determine the limit of a composite trigonometric function as the variable approaches infinity. Solving this problem necessitated a sophisticated combination of algebraic manipulation and conceptual reasoning. Figure 4 illustrates the specific steps and working provided by one of the students.

Figure 4

Response to question 2(b) by Student 39



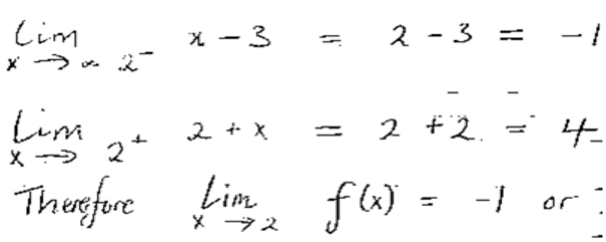
Student 39 was unable to complete all the actions necessary to reach the correct solution. While there was some evidence of interiorized actions during the initial stage, the student's grasp of limits for these specific trigonometric functions remained at the Action level. They had not yet fully interiorized these actions into stable mental processes. Furthermore, the researchers identified a specific misconception: Student 39 treated the infinity symbol as a discrete number that could be used for direct substitution.

In item [c_i], only one student failed to provide the correct answer. The solution required a simple substitution of 2 for x in the expression f(x). While a student with an Action understanding should have been able to perform this, the student in question substituted the value into the incorrect function, computing the left-hand limit instead of the right-hand limit. This error suggests the student's understanding of the limit concept had not progressed beyond the Action stage.

Finally, Question [c_{ii}] required higher-level mental processes to determine whether a limit existed, specifically utilizing the principle of the uniqueness of a limit. Approximately 27.5% of students failed to identify that the limit did not exist at the given point. This indicates they had not yet attained a Process understanding of limits when dealing with split (piecewise) functions of this nature. A representative student response is illustrated in Figure 5.

Figure 5

Response to question 2c (ii) by Student 41



Student 41 successfully computed both the left-hand and right-hand limits of the function, demonstrating at least an Action level of understanding. However, the student failed to recognize that the one-sided limits were not equal, which is necessary for a general limit to exist.

Meanwhile, ten students (19.6%) provided correct answers for all items in Question 2. These students were categorized as having developed at least an Object understanding of the limit concept for the functions tested, as they could treat the limit as a distinct mathematical entity rather than just a computational procedure. Therefore for those students there was evidence that the students had developed the limit schema.

Discussions

The results of this study revealed that while most undergraduate mathematics students attained an Action understanding of the limit concept, fewer than half exhibited an Object understanding. Surprisingly, some students failed to reach even the action level for the specific functions tested. Empirical evidence suggests these students struggled primarily with determining limits for composite and trigonometric functions, indicating they had not yet interiorized the necessary actions into mental processes. This finding directly supports the work of Maharaj (2010) in the South African context, confirming that when the external stimulus is modified from a basic polynomial to a more complex algebraic structure, the Action schema fractures. Students who rely purely on memorized algebraic manipulation (Sebsibe et al., 2019) or who struggle to decode the notation of composite functions (Fernandez, 2004) are stopped at the gate; they cannot interiorize these actions into mental processes. Without this interiorization, analyzing the behavior of $f(x)$ as $x \to a$ remains an un-executable mental operation.

A prevalent issue identified in student responses was the conflation of functional continuity at a point a with the existence of a limit at that same point. Many students asserted that if a function is discontinuous at a, the limit must automatically be zero or non-existent.

From an APOS perspective, this indicates a failure to coordinate two distinct mental structures: the *Process* of evaluating the neighborhood behavior ($x \to a$) and the *Action* of direct substitution ($f(a)$). Students are prematurely encapsulating these separate entities into a single, flawed structural object. This finding extends the observations of Denbel (2014) by demonstrating that this over-generalization is not just an algebraic shortcut, but a structural deficiency in the student's *Limit Schema*, where the boundary between a limit's existence and a function's defined value has been completely erased.

Our study provides strong empirical backing for Fernandez's (2004) assertion that students struggle heavily with the symbols ($\infty, L, \to$) associated with limits. We observed students treating the infinity symbol (∞) as a discrete real number, substituting it directly into algebraic expressions as if it were an operational constant.

In terms of the APOS theory, treating ∞ as a number represents a premature encapsulation: students are attempting to turn an unbounded dynamic *Process* into a static *Object* without the underlying mathematical structure to support it.

Conversely, when encountering a limit that did not exist, a distinct group of students assigned it a value of 0. Here, zero was not treated as a coordinate on the real number line, but as a cognitive placeholder for "nothingness" or the absence of a result. This extends Juter's (2005) critique of rote memorization; it proves that even when students memorize algebraic steps, their spontaneous concept images completely override formal logic when handling non-numeric outcomes.

Perhaps the most significant contribution of this data lies in how it intersections with the formalization steps outlined by Cottrill et al. (1996) and Swinyard and Larsen (2012). Cottrill et al.'s genetic decomposition posits a clean, multi-step transition where students coordinate the process of $x \to a$ with $f(x) \to L$.

However, our findings reveal that students do not naturally progress along this path when moving toward the formal epsilon-delta ($\epsilon - \sigma$) definition. Instead, our findings support Swinyard and Larsen's (2012) discovery of the "delta (σ)-first" reluctance. When explaining their work, students consistently tried to control the domain input (x approaching a) rather than starting with the challenge of the codomain interval (ϵ neighborhood around L).

While previous literature theorized why students lack certain mental structures (Maharaj, 2010), our primary artifacts explicitly confirm that the transition from a *Process* conception to an *Object* conception is bottlenecked by the student's inability to reverse this directional thinking. Students cannot operationalize the formal definition because they view epsilon and delta as symmetrically co-dependent variables within a dynamic domain, rather than recognizing the strict functional dependency of delta upon epsilon.

By using the ACE teaching-learning cycle, we can pinpoint exactly where the cognitive friction occurs: during classroom discussions (C), students can verbally describe a dynamic limit, but during individual exercises (E), they fail to translate this into the static object structures required for formal proof writing.

Conclusion

Based on these findings, it is concluded that fewer than half of the participants attained an Object understanding of limits. Furthermore, 3.9% of students failed to reach an Action understanding. These deep-seated misconceptions regarding the nature of functions and mathematical symbols remain the primary barrier to mastering the limit concept.

To address these gaps, mathematics facilitators should prioritize the eradication of these specific misconceptions through the following strategies:

- Diverse Representations: Rather than relying solely on algebraic manipulation, instructors should utilize graphical and computer-based manipulative software. Representing functions in multiple formats allows students to build more robust "concept images" and caters to visual learning styles.

- Constructivist Approaches: Facilitators should encourage students to construct their own definitions by exploring mathematical representations. This helps unpack abstract symbols, such as ∞, making them more accessible.
- Scaffolded Pacing: Given that the limit is a prerequisite for higher-level calculus, the curriculum should follow a "simple-to-complex" sequence. Students require ample time to master limits of simple linear functions before progressing to trigonometric or composite functions.

- The ACE Framework and Collaborative Learning: While the ACE (Activities, Class discussion, Exercises) approach is effective, it requires more time for student interaction. Peer discussions held before, during, and after lessons allow students to cross-examine their ideas, reflect on their thinking, and facilitate the internalization of actions into mental processes.

The researchers recommend further longitudinal research into the efficacy of these collaborative teaching approaches in the context of limit theory.

Acknowledgements

The researchers wish to express their sincere gratitude to all the students who participated in this study; their cooperation and insights were essential to its completion.

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Corresponding Author Contact Information:

Author name: Edmore Mangwende

Department: Mathematics and Computational Sciences

University, Country: University of Zimbabwe, Zimbabwe

Email: mangwendeeddy@gmail.com

Please Cite: Mangwende, E., & Maharaj, A. (2026). An APOS Analysis of Undergraduate Mathematics Students' Understanding of a Limit of a Function Concept: A Case of a University in Zimbabwe. *Journal of Research in Science, Mathematics and Technology Education, 9* (3), 1-15. https://doi.org/10.31756/jrsmte.931

Conflict of Interest: The authors declare that there are no conflicts of interest regarding the research, authorship, or publication of this article. This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Data Availability Statement: All data are available from the authors.

Ethics Statement: All the participants were informed of their roles in the study and they completed a consent form. Permission to carry out the study was sought from the Department of Mathematics and Computational Sciences at the University of Zimbabwe.

Author Contributions: Edmore Mangwende: Main idea, conceptualization, collecting data, analysis, reporting and writing the manuscript.
Aneshkumar Maharaj: Reviewing, proofreading and validation.

Received: March 06, 2026 ▪ Accepted: July 07, 2026